SHORT LIST OF TOPICS FOR MATHEMATICS EXAM

FOR ADMISSIONS TO NES ASPIRANTURA PROGRAM

The mathematics exam consists of two parts, each 90 minutes long.

The first part of the exam will contain 16 multiple choice questions, and 2 true-false problems, which are similar to those in the entrance examination for the MAE program. A full list of topics for this part of the exam may be found in Sections 1.1-1.4 of the official examination program at this link:

https://www.nes.ru/files/ADMISSIONS/admissions2024/matematika-apirantura.pdf

Sample questions for the first part of the exam can be found at this link:

https://www.nes.ru/files/ADMISSIONS/admissions2024/exams2000-2023.pdf

Note, however, that all questions in the entrance exam for the Aspirantura Program will be in English.

The second part of the exam will consist of (open ended) essay questions based on MAE courses Mathematics for Economists I-II and Probability Theory. Pages below contain short lists of topics for this part of the exam and some lecture notes.

MATHEMATICS FOR ECONOMISTS I

The lecture numbers below refer to Prof. Evren's lecture notes, which can be downloaded here:

https://drive.google.com/drive/folders/1eBMo5QQQjfISQ4xRuusV0eNiTV-MlpQr?usp=sharing

Topic	Lecture
Optimization problems with inequality constraints, Kuhn-Tucker theorem	4, 5
Concave, convex functions, and their role in optimization	6, 7
Quasi-concave, quasi-convex functions, and their role in optimization	8
Upper and lower hemicontinuous correspondences, Kakutani fixed point theorem, maximum theorem	9, 10

Further examples and sample problems may be found in Sundaram,[1] Chapters 6-9.

Expect optimization problems with inequality constraints, and questions that demand examples, simple proofs, and knowledge of the fundamental theorems and concepts, including constraint qualifications.

[1] Sundaram, R.K., *A First Course in Optimization Theory*, Cambridge University Press, 1996.

MATHEMATICS FOR ECONOMISTS II

The lecture numbers below refer to Prof. Evren's lecture notes, which can be downloaded here:

https://drive.google.com/drive/folders/1XTN9vs27aLCGx_sVSMyR9SSPOrCGMhyL?usp=sharing

Topic	**Lecture**
Contraction mappings, the contraction mapping theorem as a means of establishing properties of a value function	0C
Deterministic dynamic programming : Bellman equation, stationary solutions, guess and verify method, Euler equations, transversality conditions,	1, 2, 4, 5, 6
Finite Markov chains : Stationary (invariant) distributions, global stability, ergodic sets, law of large numbers	8, 9
Stochastic dynamic programming : Bellman equation, stationary solutions, stochastic Euler equations, transversality conditions	10, 11, 12

Expect questions that ask you to setup the Bellman equation associated with a stationary dynamic optimization problem, and/or solve the problem based on the guess and verify method or Euler equations. On Markow chains, simple calculations and true-false questions based on lecture notes are likely to appear.

For further clarification and examples, you may refer to Stokey and Lucas,[2] Chapters II and III.

[2] Stokey, N.L. and Lucas, R.E., with Prescott, E.C., *Recursive Methods in Economic Dynamics*, Harvard University Press, 1989.

PROBABILITY THEORY

Chapter numbers refer to the following textbooks:

(R) Ross, S.M., *A First Course in Probability*, 8th Edition, Pearson, 2010.

(S) Shiryaev, A.N., *Probability*, 2nd Edition, Springer, 1995.

Topic	**Chapter**
Sample space, event, algebra	S I.1, II.1
σ-algebra, Borel sets in R^n and R^∞	S II.1, II.2
Probability (measure), probability space	S I.1, II.1
Conditional probability, independence	S I.3
Random variables, distribution functions, independent random variables	S I.4, II.4
Expectation, variance, covariance, and correlation of discrete and continuous random variables	R 4.3-4.5, 5.2, 7.4
Conditional distributions of discrete and continuous random variables	R 6.4, 6.5
Binomial, Poisson, Normal distributions	R 4.6, 4.7, 5.4, 6.3.3, 7.8
Convergence in probability, almost sure convergence, weak convergence (convergence in distribution) and its characterizations	S II.10, III.1
Law of large numbers, central limit theorem	R 8.1-8.4

Expect questions that demand calculations, examples, simple proofs, and familiarity with the fundamental concepts and theorems.