Mathematics for Economists
module 1, academic year 2020–2021

Sergei Golovan
New Economic School
sgolovan@nes.ru

TA: Igor Samsonov (isamsonov@nes.ru)

Course description

The course "Mathematics for Economists" is designed to introduce the students to mathematical and programming tools which are widely used in economics, particularly in micro- and macroeconomics lecture courses. The course is an elective one, and is taught at the first module of the first year. It consists of 14 lectures and 7 seminars.

Course requirements, grading, and attendance policies

The course doesn't have any special prerequisites except for the standard calculus and linear algebra courses.

There will be 5 home assignments which will constitute 20% of the final grade. The final exam will account for the remaining 80%.

Course contents

1. Preliminaries
   (a) The intermediate and mean value theorems
   (b) The inverse and implicit function theorems
   (c) The basics of discrete dynamic systems
   (d) The basics of ordinary differential equations
2. Finite-dimensional optimization
   (a) Unconstrained optimization problem
   (b) Equality-constrained optimization problem, theorem of Lagrange
   (c) Unequality-constrained optimization problem, theorem of Kuhn and Tucker
   (d) Convexity and optimization
3. Parametric optimization and comparative statics

   (a) Differential comparative statics
   (b) Monotone comparative statics
   (c) Continuous comparative statics

4. Dynamic programming, Bellman's equations
   (a) Finite horizon dynamic programming
   (b) Stationary discounted dynamic programming

5. Multicriteria optimization. Pareto optimum. Application to simple games.

6. Fixed point theorems and existence of equilibria
   (a) Fixed points of contraction mappings
   (b) Fixed points of continuous mappings

Description of course methodology

Lectures will proceed from motivating examples and sample models in economics to general principles of mathematical modeling.

Sample tasks for course evaluation

1. Find the set of all solutions in the following optimization problem as a function of $w > 0$:

$$\max x^{1/4}y^{1/4} + z \quad \text{s.t. } w - x - y - z \geq 0, \ x, y, z \geq 0.$$

2. Solve the following optimization problem as a function of $\alpha > 0$:

$$\max \alpha x + y \quad \text{s.t. } y + (x - 1^3) \leq 0, \ x, y \geq 0.$$

3. Consider the problem

$$\begin{aligned} &\min \left[(x_1 - 2)^2 + (x_2 - 1)^2, x_1^2 + (x_2 - 3)^2\right] \\ &\text{s.t. } g_1(x) = x_1^2 - x_2 \leq 0 \\ &\qquad g_2(x) = x_1 + x_2 - 2 \leq 0 \\ &\qquad g_3(x) = -x_1 \leq 0 \end{aligned}$$

   (a) Find at least one candidate for a properly efficient solution $\hat{x}$ (in the sense of Kuhn and Tucker).
   (b) Try to determine all candidates.

4. Consider the following representative agent's utility maximization problem (so called centrailsed Ramsey model):

$$\begin{aligned} &\max \sum_{t=0}^{\infty} \beta^t u(c_t) \\ &\text{s.t. } k_{t+1} = f(k_t) + (1 - \delta)k_t - c_t, t = 0, 2, \ldots \\ &\qquad k_0 \text{ is given} \end{aligned}$$

Here c_t is the consumption, k_t is the per capita capital, $\beta, \delta \in (0,1)$, the utility function

$$u(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma},\ \gamma > 0,$$

(if $\gamma = 1$ then $u(c_t) = \ln c_t$, γ is the coefficient of risk aversion), the production function

$$f(k_t) = Ak_t^{\alpha},\ A > 0,\ \alpha \in (0,1).$$

(a) Rewrite this problem as a stationary dynamic programming problem, i. e. specify the S, A, r_t, f_t, Φ_t.

(b) Denote $V(k)$ the maximum value of the utility function given the starting value of capital $k_0 = k$ and write out the Bellman equation for the problem.

(c) Draw the optimum paths of investment and consumption for three values of α: 0.3, 0.5, 0.7 at the same graph (choose all the other parameters at your discretion).

(d) Draw the optimum paths of investment and consumption for three values of γ: 0.5, 1, 2 at the same graph (choose all the other parameters at your discretion).

Course materials

1. Rudin, W. (1976) Principles of Mathematical Analysis, Third Edition, McGraw-Hill International Editions, Singapore.

2. Sundaram R. K. (1996), A First Course in Optimization Theory, Cambridge University Press.

3. Ljungqvist L., T. J. Sargent (2012) Recursive Macroeconomic Theory, Third Edition, MIT Press.

4. Ehrgott M. (2005) Multicriteria Optimization, Second Edition, Springer.

5. Osborne, M. J. (2003) An Introduction to Game Theory, Oxford University Press.

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.