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Large Bets and Stock Market Crashes

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Abstract

Some market crashes occur because of significant imbalances in demand and supply. Conventional models fail to explain the large magnitudes of price declines. We propose a unified structural framework for explaining crashes, based on the insights of market microstructure invariance. A proper adjustment for differences in business time across markets leads to predictions which are different from conventional wisdom and consistent with observed price changes during the 1987 market crash and the 2008 sales by Société Générale. Somewhat larger-than-predicted price drops during 1987 and 2010 flash crashes may have been exacerbated by too rapid selling. Somewhat smaller-than-predicted price decline during the 1929 crash may be due to slower selling and perhaps better resiliency of less integrated markets.

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After stock market crashes, market participants, policymakers, and economists are usually unable to explain what happened. Heavy selling pressure has been recorded during crashes. It is known that large imbalances move prices in the direction of trades, as discussed by Kraus and Stoll (1972), Grinold and Kahn (2000), and Gabaix (2009), but there is no compelling quantitative explanation for why the seemingly small quantities sold might have led to such large price dislocations in the highly liquid stock market.

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We investigate this issue through the lens of market microstructure invariance, a conceptual framework developed by Kyle and Obizhaeva (2016). By analyzing prices and quantities in market-specific "business time," defined as time which is measured by counting market events rather than seconds on a clock, we are able to explain why seemingly small observed imbalances in demand and supply can indeed have created such large market crashes.

We illustrate our approach by studying five crash events, chosen because data on the magnitude of contemporaneous selling pressure became publicly available in their aftermaths:

- After the stock market crash of October 1929, the Senate Committee on Banking and Currency (1934) (the "Pecora Report") documented the plunge in broker loans attributed to forced margin selling during the crash.
- After the October 1987 stock market crash, the U.S. Presidential Task Force on Market Mechanisms (1988) (the "Brady Report") reported quantities of stock index futures contracts and baskets of stocks sold by portfolio insurers during the crash.
- After the futures market dropped by 20% at the open of trading 3 days after the 1987 crash, the Commodity Futures Trading Commission, Division of Economic Analysis and Division of Trading and Markets (1988) identified large sell orders executed at the open of trading; the press identified the seller as George Soros.
- After the Fed cut interest rates by 75 basis points in response to a worldwide stock market plunge on January 21, 2008, Société Générale revealed that it had been quietly liquidating billions of Euros in stock index futures positions accumulated earlier by rogue trader Jérôme Kerviel.
- After the flash crash of May 6, 2010, the Staffs of the CFTC and SEC (2010a, 2010b) cited as its trigger the large sales of futures contracts by one entity, later identified in the press as Waddell & Reed.

We do not study the flash crash events in 1961 and 1989, the collapse of Long Term Capital Management in 1998, the quant meltdown of August 2007, or the U.S. Treasury note flash rally in October 2016 because data on the size of sales which precipitated these events are not available.

Each of the five crashes is associated with a large sell bet, where we think of a "bet" (or "meta-order") as being a statistically independent decision either to speculate on information or to hedge risks by buying or selling significant quantities of risky financial assets, often implemented as sequences of orders executed over time. These bets resulted either from trading by one entity or from correlated trading of multiple entities with the same underlying motivation.

Many practitioners and academics believe that during these five crashes, selling pressure was too small to induce significant price declines. We call this interpretation "conventional wisdom." Scholes (1972), Harris and Gurel (1986), and Wurgler and Zhuravskaya (2002) illustrate conventional wisdom by claiming that the demand for financial assets is elastic in the sense that selling 1% of the asset's market capitalization has a price impact of less than 1%. When applied to market bets during the five crashes, this conventional thinking implies tiny market impact. Indeed, given typical turnover rates of stocks, this implies that sales of 5% of average daily volume are expected to have only modest impact on stock prices. Naively extrapolating these estimates to the highly liquid market for stock index futures contracts further suggests that selling 5% of daily volume of the entire stock market must have even smaller impact on the overall level of stock prices.

Microstructure invariance implies a different way to extrapolate price impact estimates from stocks to index futures. This approach implies much larger price impacts of bets in liquid futures markets because it models trading in market-specific business time, not calendar time.

The main intuition is as follows. As we show below, invariance principles imply that business time passes about 225 times faster in the equities market as a whole than in markets for less liquid individual stocks. One calendar day of trading in stock index futures is thus equivalent to 225 calendar days of trading in a stock. Hence, a sell bet of 5% of 1 day's volume for index futures is analogous to selling 5% of daily volume each day for 225 consecutive calendar days (not 1 day) for a stock or a bet of 1125% ($= 5\% \times 225$) of 1 day's volume. A bet of 5% of 1 day's volume for index futures therefore must have much bigger (not smaller) price impact than a bet of 5% of 1 day's volume for less liquid individual stocks. This argument goes against the standard intuition.

We calculate price impacts implied by quantities traded during crashes using a universal formula for market impact, as suggested by invariance. The price impact is a function of the dollar size of a bet, expected dollar volume, returns volatility, and a couple of invariant parameters. Kyle and Obizhaeva (2016) calibrate these parameters using a database of about 400,000 portfolio transition orders executed during the period 2001–05 in US stocks. Portfolio transition orders are well suited for calibration of market impact functions because they can be thought of as exogenous shocks to demand and supply. In this article, we extrapolate the estimates from the sample of relatively small individual US stocks to the large US stock market as a whole and from 2001 to 2005 to other historical periods. Except for the time frame of execution, the spirit of invariance suggests that institutional details related to market structure, information asymmetries, or motivation of traders should not affect market impact estimates much. We show that the implied estimates are indeed large enough to explain crashes.

Table I summarizes our results for each of the five crash events. The table shows the actual percentage decline in market prices, the percentage decline predicted by invariance, the percentage decline predicted by conventional wisdom, the dollar amount sold as a fraction of average daily volume, and the dollar amount sold as a fraction of 1 year's GDP.

Table I. Summary of five crash events

Actual and predicted price declines. The table columns show the actual price changes, predicted price changes, and bets as percent of average daily volume and GDP. The predictions implied by invariance are much closer to actual price changes than those implied by conventional wisdom.

Crash Event	Actual (%)	Predicted invariance (%)	Predicted conventional (%)	%ADV	%GDP
1929 market crash	25	46.43	1.36	265.41	1.136
1987 market crash	32	16.77	0.63	66.84	0.280
1987 Soros's trades	22	6.27	0.01	2.29	0.007
2008 SocGén trades	9.44	10.79	0.43	27.70	0.401
2010 flash crash	5.12	0.61	0.03	1.49	0.030

We assume that the market impact of a bet is equal to the percentage of market capitalization sold when calculating the estimates implied by conventional wisdom.

For all events, the price impact estimates based on conventional wisdom are minuscule in comparison to actual price changes. Other estimates obtained from the analysis of trades of institutional investors—such as studies by Grinold and Kahn (2000), Torre (1997), Almgren *et al.* (2005), and Frazzini, Israel, and Moskowitz (2018)—produce estimates higher than conventional intuition but far too small to explain crashes (reported later). In contrast, predictions based on invariance are broadly similar to actual price declines.

Across the five crash events, there are substantial differences between actual declines and predictions based on invariance. The predictions of invariance do not take into account the speed with which sales took place or the fragility of markets at the times of the crashes.

At the same time, the speed of sales may have influenced the magnitudes of price declines. In 1929, efforts were made to spread the impact of margin selling out over several weeks rather than several days. This may have made the actual price decline of 25% smaller than the forecast of 46.43%. The 1987 Soros trades and the 2010 flash crash were both "flash-crash" events in which prices declined rapidly and then recovered within minutes. The unusually rapid rate at which bets were executed may have magnified temporary price impact.

Variation in market integration across assets, availability of capital available to take the other sides of large sales, and disruptions to the market mechanism may also help explain differences between actual and predicted price declines. In 1929, smaller-than-predicted price declines may have been reduced by markets being less integrated than today. Potential buyers were keeping capital on the sidelines to profit from price declines widely expected to occur if margin purchases were liquidated. In 1987, larger-than-predicted price declines may have been exacerbated by breakdowns in the market mechanism documented in the Brady Report.

These five bet-induced crashes differ from macroeconomic crises with sovereign defaults, bank failures, exchange rate collapses, and bouts of high inflation catalogued by Reinhart and Rogoff (2009). Recovery from economic crises in contrast takes many years, even after significant changes in macroeconomic policies and market regulation. Bet-induced crashes are likely to be short-lived, especially if followed by appropriate government policy. For example, the looser monetary policy implemented by Federal Reserve System immediately after the 1929 crash calmed down the market by the end of 1929. Even though the wealth effect of declining equity prices may have helped trigger a recession by reducing consumption, Friedman and Schwarz (1963) write that the Great Depression of the 1930s resulted from a subsequent shift toward a deflationary monetary policy, not from the 1929 crash itself. Similarly, the unwinding of Jérôme Kerviel's large rogue bet in January 2008 was followed by the collapse of Bear Stearns a few weeks later, but the deep and long-lasting recession which unfolded in 2008–09 was triggered by the bursting of the real-estate credit bubble, not from liquidation of his bet.

Unable to find rational quantitative explanations, some researchers believe that market crashes result from irrational behavior. The "animal spirits" hypothesis of market crashes says that price fluctuations occur as a result of random changes in psychology and emotions, which may not be based on economically relevant information or rational calculations. Keynes (1936) said that financial decisions may be taken as the result of "animal spirits—a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities." Akerlof and Shiller (2009) echo Keynes (1936): "To understand how economies work and how we can manage them and prosper, we must pay attention to the thought patterns that animate people's ideas and feelings, their animal spirits." Promptly after the 1987 crash, Shiller (1987) surveyed traders and found that "most investors interpreted the crash as due to the psychology of other investors."

In contrast, we believe that one does not need to invoke irrationality to explain crashes. Before the 1929 crash, market participants widely discussed the possibility that forced liquidations of margin accounts would lead to a collapse in prices. Before the 1987 crash, market participants discussed that portfolio insurance sales might lead to a market meltdown. In the absence of quantitative justification, these prescient views were largely dismissed due to a deeply entrenched ideological belief that the demand for equities is elastic. Here, we provide quantitative justification for a theory of crashes based on the market impact of large bets, not based on psychology. The margin sales of 1929, portfolio insurance sales of 1987, and liquidation of Kerviel's rogue positions were all large bets resulting from the rapid execution of mechanical trading strategies, not from psychology. While the sales of George Soros in 1987 and sales of Waddell and Reed in 2010 may reflect the animal spirits of one person and one entity, the immediate recovery of prices suggests the opposite of market-wide irrationality or psychological contagion.

The remainder of this article discusses the conventional wisdom in assessing market impact, market microstructure invariance, particulars of each of the crash events, and lessons learned.

1. Market Impact of Large Bets: Previous Literature

Previous studies have used different methodologies to obtain widely varying estimates of the market impact of large bets. This literature can be divided into two strands. The first strand, which we call "conventional wisdom," examines the price effects of seasoned equity offerings, changes in the composition of the S&P 500 index, and similar events. The second strand examines the price impact of trades by institutional investors.

1.1 Conventional Wisdom

Scholes (1972) claimed that the price impact of large sales of equities is negligible based on his analysis of secondary equity distributions. Harris and Gurel (1986), Wurgler and Zhuravskaya (2002), and others study the price response to additions and deletions of stocks to equity indices like the S&P 500 and infer that selling 1% of an individual stock's shares outstanding has a price impact of at most 1%. Wurgler and Zhuravskaya (2002, Table IV, p. 603) provide a summary of demand elasticities from different papers, all of which suggest an elastic demand for stocks; estimated demand elasticities vary from 1 to 3,000, an almost infinite elasticity from Scholes's study.

When extrapolated from the market for individual stocks to the stock market as a whole using the same demand elasticity, these empirical studies support the conventional wisdom that observed selling pressure could not have created stock market crashes.

From a theoretical perspective, the conventional wisdom is based on the logic of perfectly competitive capital markets, the capital asset pricing model, and the efficient markets hypothesis. The market risk premium of 5–7% per year reflects compensation for bearing the risk of the entire stock market for 1 year. When large bets are executed, market participants taking the other side of these bets are exposed to risks of much smaller magnitude than the entire stock market and hold positions over much shorter horizons than 1 year, usually a few days or minutes. Hence, the compensation required for absorbing large bets should be dramatically smaller than the equity market risk premium.

Indeed, when conventional wisdom was applied to the crash of 1987, prominent financial economists claimed that the price impact of reported sales was up to 100 times too small to generate a crash. Leland and Rubinstein (1988), the academics most closely associated with portfolio insurance in 1987, say, "To place systematic portfolio insurance in perspective, on October 19, portfolio insurance sales represented only 0.2% of total U.S. stock market capitalization. Could sales of 1 in every 500 shares lead to a decline of 20% in the market? This would imply a demand elasticity of 0.01—virtually zero—for a market often claimed to be one of the most liquid in the world." Miller (1991) makes similar claims about the 1987 crash: "Putting a major share of the blame on portfolio insurance for creating and overinflating a liquidity bubble in 1987 is fashionable, but not easy to square with all relevant facts. … No study of price–quantity responses of stock prices to date supports the notion that so large a price decrease (about 30%) would be required to absorb so modest (1%–2%) a net addition to the demand for shares." Using a calibrated theoretical model of competitive capital markets, Brennan and Schwartz (1989) claimed that portfolio insurance sales (of 0.63% of market capitalization) in the 1987 crash would have a price impact of about 100 times smaller than the 32% price drop observed.

Since price pressure was thought to be too small to explain quantitatively market crashes, some observers of the 1987 stock market crash, including Miller (1988, p. 477) and Roll (1988), sought to explain the large price declines as market reactions to new fundamental information rather than response to trading, but it was difficult to find new information to which market prices would have reacted so dramatically.

In our analysis below, we summarize conventional wisdom using the least conservative conventional estimate and assume a unit demand elasticity for stocks: Selling 1% of capitalization moves prices down by 1%. Mathematically, suppose a stock's price is P, outstanding shares are N, and shares sold are Q. Then, the expected log-percentage market impact $\Delta \ln P$ is Q/N:

$$\Delta \ln P \approx \frac{\Delta P}{P} = \frac{Q}{N}. \tag{1}$$

Throughout this article, we adopt the convention that Q is the unsigned trade size and $\Delta P/P$ is the expected unsigned price impact.[1]

The conventional market impact function can be also expressed in terms of average daily volume. Let V denote daily volume in shares. Assume for simplicity that an asset's turnover is approximately 100% per year with 250 trading days. Since 1% of capitalization is approximately equal to 250% of daily volume, the conventional wisdom [Equation (1)] can be interpreted as

1 The size of market impact $\Delta \ln P$ is either the expectation of the post-trade log-price minus pre-trade log-price for buy bets or the expectation of the pre-trade log-price minus post-trade log-price for sell bets. A similar formula can be written for simple percentage impact $\Delta P/P$, where ΔP is either the difference between post-trade price and pre-trade price for buy bets or the difference between pre-trade price and post-trade price for sell bets.

$$\Delta \ln P \approx \frac{\Delta P}{P} = \frac{Q}{250 \text{ days } \cdot V}. \tag{2}$$

The Brady Report used this intuition to compare daily volume elasticities in the 1987 crash to the 1929 crash:

> To account for the contemporaneous 28% decline in price, this implies a price elasticity of 0.9 with respect to trading volume which seems unreasonably high. As a percentage of total shares outstanding, margin-related selling would have been much smaller. Viewed as a shift in the overall demand for stocks, margin-related selling could have accounted realistically for no more than 8% of the value of outstanding stock. On this basis, the implied elasticity of demand is 0.3 which is beyond the bound of reasonable estimates.

1.2 Estimates from Institutional Trades

The finance literature which studies price responses to large institutional bets typically finds that the demand for stocks is less elastic than what conventional wisdom suggests.

Kraus and Stoll (1972) study block trades of large NYSE stocks. More recent estimates of market impact from executions of large orders by institutional investors include Chan and Lakonishok (1995, 1997) and Keim and Madhavan (1997).

Some studies find nonlinear price impact. The "square root model" or Barra model, described by Grinold and Kahn (2000) and Torre (1997), says that the execution of an order of size Q on average moves price by $\Delta P/P = 1 \cdot \sigma \cdot (Q/V)^{1/2}$. The square root model implies that an order for 25% of 1 day's volume in a stock with 2% daily volatility implies a price impact of 100 basis points. With 100% annual turnover, this implies an elasticity of demand of 0.10.

Frazzini, Israel, and Moskowitz (2018) estimate a more complicated version of the square root model. Almgren *et al.* (2005) incorporate execution horizon into a model with concave price impact similar to a square root model.

While less elastic demand from these models implies more price impact than conventional wisdom, a demand elasticity of only 0.10 is nevertheless not small enough to generate crashes; this is shown in Appendix B, which extrapolates the estimates of these models to crash events. For crashes to result from selling pressure, the elasticity needs to be approximately 0.01, yet another order of magnitude more inelastic than implied by these studies. The square root model makes it especially difficult to explain crashes because its implied concave price impact makes marginal price impact decrease as the size of large bets increases. To explain crashes, we use a linear price impact model, which is popular with finance theorists because it excludes simple forms of arbitrage (Huberman and Stanzl, 2004).

2. Market Impact of Large Bets: Invariance

Invariance implies an alternative methodology for extrapolating price impact from the less liquid markets for individual stocks to the more liquid market for indexes of all stocks. While we agree with demand elasticities estimated for individual stocks in studies of institutional trades, we suggest an alternative approach for how to extrapolate these estimates to the stock market as a whole. By taking into account differences in business time, invariance can explain stock market crashes.

2.1 Review of Invariance

Invariance is based on the simple intuition that trading in a speculative market is a game in which financial risks are exchanged in business time. The speed of business time varies significantly across assets. It is proportional to the rate at which new bets—or trading ideas—arrive. Trading is fast in liquid markets and slow in illiquid markets.

Invariance consists of two conjectures: (1) The distribution of standard deviations of dollar gains and losses on bets is the same across markets, *when standard deviation is measured in units of business time.*[2] (2) The expected dollar costs of executing equivalent bets are constant across markets, *when equivalent bets are defined to transfer the same dollar risks per unit of business time*. These invariance conjectures imply specific scaling laws for financial variables.

We derive these scaling laws using a simplified version of Kyle and Obizhaeva (2016).[3] Kyle and Obizhaeva (2020) show how to obtain the same scaling laws in the context of an equilibrium model of speculative trading with endogenous acquisition of private information and endogenous entry into the market. In this model, scaling laws for bet sizes and transaction costs ultimately follow from the assumption that the effort required to generate private signals does not vary across markets. This is likely to hold, at least approximately, in an equilibrium where traders allocate their skills optimally across markets.

2.2 Business Time

For a given stock, suppose that bets of average size $\overline{Q}$ arrive at rate γ. For a typical stock, we might have $\gamma = 100$ bets per day and $\overline{Q} = 10,000$ shares. As γ increases, market participants transfer risks more quickly and business time passes at a faster rate relative to calendar time.

Since individual bets are difficult to observe, it is hard to measure γ and $\overline{Q}$ in practice. Nevertheless, the invariance hypotheses make it possible to infer γ and $\overline{Q}$ from daily dollar volume $P \cdot V$ and daily returns volatility σ up to a constant which is the same for all stocks. The proof is based on two simple equations.

First, define trading activity W as the product of dollar volume and returns volatility:

$$W := P \cdot V \cdot \sigma. \tag{3}$$

Trading activity better reflects the rate at which the market transfers risks than dollar volume $P \cdot V$ because it takes into account that trading assets with higher volatility σ transfers more risk per dollar traded. Since all bets sum up to volume, $V = \gamma \cdot \overline{Q}$, we can write W in terms of γ and $\overline{Q}$:

$$W = \gamma \cdot \overline{Q} \cdot P \cdot \sigma. \tag{4}$$

Since dollar volume $P \cdot V$ has units of dollars/day and returns volatility σ has units per day$^{1/2}$, trading activity W has units dollars/day$^{3/2}$.

2 This conjecture does not say that dollar returns volatility or returns volatility are constant in business time.

3 Kyle and Obizhaeva (2018b) obtain similar predictions using dimensional analysis and leverage neutrality. Kyle and Obizhaeva (2018a) derive them from a meta-model, a system of simple equations inherent to many microstructure models. Kyle, Obizhaeva, and Wang (2018) provide illustration using a one-period equilibrium model.

Second, the first invariance conjecture says that dollar risk $P \cdot \sigma$ transferred by an average bet of $\overline{Q}$ shares per unit of business time $1/\gamma$ is invariant across markets. Thus, for some dollar constant $\overline{C}$, such as $\overline{C} = \$2000$, we have

$$\overline{Q} \cdot \frac{P \cdot \sigma}{\sqrt{\gamma}} = \overline{C}. \tag{5}$$

Equations (4) and (5) make up a system of two log-linear equations in two unknowns $a := P \cdot \overline{Q} \cdot \sigma$ and γ:

$$a \cdot \gamma = W \qquad \text{and} \qquad a \cdot \gamma^{-1/2} = \overline{C}. \tag{6}$$

The solution for a and γ is

$$a = \overline{C} \cdot \left(\frac{W}{\overline{C}}\right)^{1/3} \qquad \text{and} \qquad \gamma = \left(\frac{W}{\overline{C}}\right)^{2/3}. \tag{7}$$

Now define $H := 1/\gamma$ as the time interval between bets. For example, if $\gamma = 100$ bets per day for some stock, then H is about 4 min during trading hours from 9:30 a.m. to 4:00 p.m. Equation (7) implies that average dollar bet size $P\overline{Q}$ and time between bets H are given by

$$P \cdot \overline{Q} = \frac{\overline{C}}{\sigma} \cdot \left(\frac{W}{\overline{C}}\right)^{1/3} \qquad \text{and} \qquad H := 1/\gamma = \left(\frac{W}{\overline{C}}\right)^{-2/3}, \quad \text{where} \qquad W := P \cdot V \cdot \sigma. \tag{8}$$

Since W has units of dollars/day$^{3/2}$ and $\overline{C}$ has units of dollars, Equation (8) has correct units of dollars for $P \cdot \overline{Q}$ and days for H. Equation (8) shows how to extrapolate the size and number of bets from one stock to another, under the invariance assumption that $\overline{C}$ is constant across markets.

Define a benchmark stock as a security with stock price $P^* = \$40$ per share, expected volume $V^* = 10^6$ shares per calendar day, expected percentage returns volatility $\sigma^* = 0.02$ per day$^{1/2}$, and trading activity $W^* = P^* \cdot V^* \cdot \sigma^*$; these parameters would approximately correspond to a stock from the bottom of the S&P 500 index. Suppose the time interval between bets for this stock is H^* minutes, say, H^* is about 4 min. Equation (8) implies that H is inversely proportional to the two-third power of trading activity,

$$\frac{1}{H} = \frac{1}{H^*} \cdot \left(\frac{W}{W^*}\right)^{2/3} = \frac{1}{H^*} \cdot \left(\frac{P \cdot V \cdot \sigma}{P^* \cdot V^* \cdot \sigma^*}\right)^{2/3}. \tag{9}$$

Business time H represents different lengths of calendar time for different assets: 4 min for the benchmark stock, an hour for thinly traded stocks, less than 1 min for actively traded stocks, and about 1 s for the market as a whole. This is derived by plugging values for volume and volatility into the equation above. Business time differs across assets.

The conventional wisdom makes the mistake of extrapolating from one market to another under the implicit assumption that business time H is constant across markets.

2.3 Distribution of Bet Size

The logic of invariance can be applied to the entire distribution of random bet sizes $\tilde{Q}$, not just the means $\overline{Q}$. This logic implies that probability distributions of bet sizes $\tilde{Q}$ must be the same across markets if $\tilde{Q}$ is scaled by trading volume per business day $V \cdot H$, rather than by trading volume per calendar day V. When bet size $\tilde{Q}$ is scaled by VH, Equation (9) implies that the resulting scaled bet size $\tilde{Z}$ has a mean of 1 and the same distribution for all stocks:

$$\frac{\tilde{Q}}{V \cdot H} \stackrel{\mathrm{d}}{=} \tilde{Z} \qquad \text{which implies} \qquad \frac{\tilde{Q}}{V} = H^{*} \cdot \left(\frac{W}{W^{*}}\right)^{-2/3} \cdot \tilde{Z}. \tag{10}$$

Equivalent bets transfer the same dollar risks in business time. In calendar time, equivalent bets correspond to a smaller fraction of daily volume in markets with larger trading activity and thus shorter time interval between bet arrivals.

2.4 Price Impact

The logic of invariance can also be extended to market impact. Think of bets in two different markets as equivalent if they have the same scaled size $\tilde{Z} = \tilde{Q}/(VH)$. Bet cost invariance conjectures that equivalent bets have the same price impact when scaled by returns volatility in business time.

Let $\Delta P/P$ denote price impact of a bet of size Q and let $\sigma\sqrt{H}$ denote volatility in business time. Bet cost invariance therefore implies an invariant price impact function $f()$ such that

$$\frac{\Delta P}{P} = \sigma \cdot \sqrt{H} \cdot f(Z), \quad \text{where} \qquad Z = \frac{Q}{VH}. \tag{11}$$

If the price impact function is modeled as a power function $f(Z) = \alpha \cdot Z^{\beta}$ with proportionality constant α and exponent β, then Equation (11) takes the form

$$\frac{\Delta P}{P} = \alpha \cdot \sigma \cdot \sqrt{H} \cdot \left(\frac{Q}{V \cdot H}\right)^{\beta}. \tag{12}$$

Plugging in H from Equation (9) and assuming linear market impact ($\beta = 1$) yields

$$\frac{\Delta P}{P} = \alpha \cdot \left(\frac{W}{\overline{C}}\right)^{1/3} \cdot \sigma \cdot \left(\frac{Q}{V}\right), \quad \text{where} \qquad W := \frac{PV\sigma}{\overline{C}}. \tag{13}$$

In comparison with conventional intuition [Equation (2)] that bets of the same fraction of daily volume Q/V must have the same percentage price impact, holding volatility σ constant, the linear specification [Equation (13)] has the additional factor $(W/\overline{C})^{1/3}$, which shows up due to the faster pace of business time in markets with higher trading activity. This factor makes the demand more inelastic as trading activity increases. For example, increasing W by a factor of 1,000 decreases the elasticity of demand by a factor of 10 and reduces demand elasticity from say 0.10 to 0.01. As we shall see, if the market as a whole is viewed as one big market with very fast business time, this makes the demand elasticity for the market as a whole low enough for observed order imbalances to explain the average size of price declines in market crashes.

2.5 Intuition behind Equivalent Bets

We next compare the magnitude of selling pressure during five market crashes with the sizes of large institutional orders executed in US equities.

As a yardstick for measuring the size of institutional orders, we use the largest orders from the 400,000+ portfolio transition orders studied by Kyle and Obizhaeva (2016). A portfolio transition occurs when assets managed by one institutional asset manager are transferred to another manager. Trades converting the legacy portfolio into the new portfolio are typically handled by a professional third-party transition manager. Portfolio transitions represent some of the largest changes in portfolios held by institutional investors during the year. We estimate the distributions of portfolio transition orders to be symmetric around zero with unsigned order sizes close to log-normal random variables with different log-means and the same log-variance of 2.53:

$$\ln\left(\frac{\tilde{Q}}{V}\right) \sim \mathcal{N}(-5.71 - \frac{2}{3} \cdot \ln\left(\frac{W}{W^*}\right), 2.53). \tag{14}$$

The empirical distribution of $\ln(\tilde{Q}/V)$ has a slope of $-2/3$ with respect to change in the log of trading activity $\ln(W)$, as expected given prediction (10).

Figure 1 summarizes the intuition of our article in one snapshot. It shows two types of extrapolations across markets; one is based on conventional wisdom and another is based on invariance. The vertical axis is the log of order size as a fraction of daily volume $\ln(Q/V)$. The horizontal axis is the log of scaled trading activity $\ln(W/W^*)$. The point $\ln(W/W^*) = 0$ corresponds to the benchmark stock with trading activity W^* from the bottom of the S&P 500 index. Trading activity varies from $\ln(W/W^*) = -12$ for the least actively traded stocks to $\ln(W/W^*) = 2.00$ for the most actively traded stocks such as Apple, by a factor of about 10^6 $(= \exp(12 + 2))$. Trading activity in the overall stock market—which includes both stocks and futures trading—is much higher, up to $\ln(W/W^*) = 8.20$; it is about 500 $(= \exp(8.20 - 2.0))$ times larger than the trading activity of the most liquid stocks.

The (black) horizontal lines show the extrapolation direction implied by the conventional wisdom. These isoquants mark orders of sizes equal to a given percentage of calendar-day volume. For example, the horizontal line $|Q/V| = 5\%$ represents orders equal to 5% of daily volume; conventional wisdom suggests that these bets will have the same impact.

The diagonal (red and green) lines with slopes of $-2/3$ show equivalent bets as implied by the theory of invariance. The lowest diagonal (red) line identifies log-medians of Q/V for different markets, as implied by Equation (14); this line intersects the vertical axis at -5.71, a point corresponding to a median bet in the benchmark stock equal to $\exp(-5.71) \cdot V$ or approximately $0.33\% \cdot V$. The six diagonal (green) parallel lines above the median line mark orders whose log-sizes are one–six standard deviations above the log-median sizes, respectively. Each log standard deviation represents an increase in bet size by a factor of $\exp(2.53^{1/2}) \approx 4.90$.

For each of the 60 months from January 2001 to December 2005, the 400,000+ portfolio transition orders are sorted into volume bins based on thresholds corresponding to the 30th, 50th, 60th, 70th, 75th, 80th, 85th, 90th, and 95th percentiles of the dollar volume for NYSE-listed common stocks. The 600 blue diamonds in Figure 1 represent the largest orders in each of 10 volume bins for each of 60 months. The diamond points form a cloud tilted along invariance-implied iso-lines with slope of $-2/3$. These dots are certainly not on a horizontal line, as would be predicted by the conventional wisdom. Since each bin contains on average about 650 points, invariance and log-normality of order size suggest that these largest portfolio transition orders should lie slightly below the 3-standard-deviation diagonal with predicted slope of $-2/3$. As can be seen visually from the figure, this is

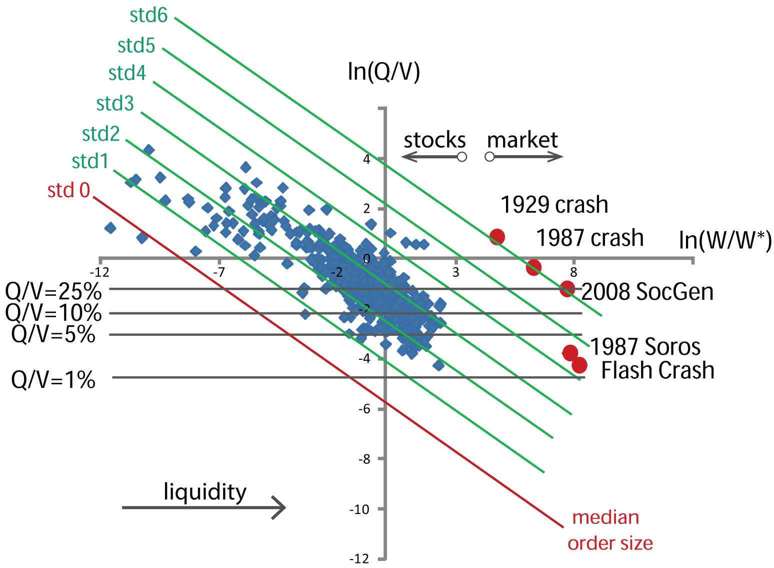

Figure 1. Largest portfolio transition orders and market crashes. This figure shows the largest portfolio transition orders for each month from January 2001 to December 2005 and for each of ten volume groups (blue points) as well as the bets during five market crashes (red points). Volume groups are based on thresholds corresponding to 30th, 50th, 60th, 70th, 75th, 80th, 85th, 90th, and 95th percentiles of dollar volume for common NYSE-listed stocks. The vertical axis is $|\ln(Q/V)|$. The horizontal axis is $\ln(W/W^*)$, where $W^* = 40 \cdot 10^6 \cdot 0.02$ and $W = P \cdot V \cdot \sigma$. The median order is $-5.71 - (2/3) \cdot \ln(W/W^*)$ (red line). The x-standard deviation events are $-5.71 - (2/3) \cdot \ln(W/W^*) + x \cdot \sqrt{2.53}$ (green lines). According to conventional wisdom, which extrapolates along horizontal lines, imbalances during the five crash events (red dots) are similar in size to typical large orders (blue diamonds). According to the invariance hypothesis, which extrapolates along lines with slope $-2/3$, imbalances during five crash events are rare outlier events.

approximately the case. The scatter plot of largest portfolio transition orders thus confirms the predictions of the invariance hypothesis.

Figure 1 also depicts the five crash events by big round red dots. Extrapolating along horizontal lines, the conventional wisdom would say that these five events are not unusual compared with the largest portfolio transition orders. These percentages of daily volume are not very different from what is seen in the largest portfolio transition orders, which are often about 25% of daily volume in liquid stocks and an even larger percentage of daily volume in illiquid stocks (up to 700%). The two flash crashes are only 2.29% and 1.49% of daily volume. Even the largest crashes—the 1929 crash, the 1987 crash, and liquidation of Kerviel's position—represent "only" 265%, 67%, and 28% of daily volume and only 1.36%, 0.63%, and 0.43% of market capitalization, respectively. Therefore, based on horizontal extrapolation, nothing unusual would be expected to happen during crash episodes.

In contrast, in the context of invariance with its extrapolation along diagonal lines with slope $-2/3$, the crash events are extremely large, even when compared with the largest institutional orders. The two flash crashes correspond to about 4.5-standard-deviation events. The 1929 crash, the 1987 crash, and the liquidation of Jérôme Kerviel's positions correspond to about 6-standard-deviation events. This suggests it would not be surprising that they caused significant price dislocations.

2.6 Invariance-Implied Market Impact Formulas

We use a log-linear version of the linear impact model [Equation (13)]. The expected percentage price impact from buying quantity Q of a security with share price P, expected daily volume V, and daily expected volatility σ is given by

$$\Delta \ln P = \frac{\bar{\lambda}}{10^4} \cdot \left(\frac{P \cdot V}{P^* \cdot V^*}\right)^{1/3} \cdot \left(\frac{\sigma}{\sigma^*}\right)^{4/3} \cdot \frac{Q}{H^* \cdot V}. \tag{15}$$

This formula assumes benchmark stock values $P^* = \$40$ per share, $V^* = 10^6$ shares per day, $\sigma^* = 0.02$ per day$^{1/2}$, with $H^* = 0.01$ days ≈ 4 min. Invariance says that this factor $\bar{\lambda}$ is the same for all markets and time periods.

Kyle and Obizhaeva (2016) estimate $\bar{\lambda}$ using data on implementation shortfall of portfolio transition orders.[4] Introduced by Perold (1988), this metric is the difference between the execution price and a "paper trading" benchmark price recorded before the order was placed. The calibrated value of $\bar{\lambda}$ is equal to 5.00, with standard error 0.38 (about 7% of the price impact). The value $\bar{\lambda} = 5.00$ is scaled so that the price impact of an order for 1% of daily volume in the benchmark stock is 5 basis points. Linear price impact therefore implies that an order for 10% of daily volume has a price impact of 50 basis points. A price impact of 50 basis points is similar to estimates in the literature on institutional block trades of similar size.

We use a log-linear version of the market impact model rather than a simple linear model because our analysis deals with very large orders, sometimes equal in magnitude to trading volume of several trading days. In contrast, Kyle and Obizhaeva (2016) consider relatively smaller portfolio transition orders with an average size of 4.20% and a median size of 0.57% of daily volume; for these smaller orders, the distinction between $e^{\pm x}$ and $1 \pm x$ is immaterial.

Equation (15) is a universal formula for market impact, which may be applied to different markets and time periods. Having calibrated it on the sample of portfolio transition orders in the individual US stocks for 2001–05, we extrapolate the same formula to large market bets and to different time periods.

In Appendix A, we discuss several implementation issues for applying invariance to the five crash events. These include defining boundaries of the market, choosing proxies for expected volume and volatility, and understanding functioning of market institutions. Appendix B reports estimates implied by a number of alternative models of market impact based on conventional wisdom and the literature on institutional trades.

4 Equation (37) of Kyle and Obizhaeva (2016, p. 1400) uses a slightly different specification, which estimates an *average* impact cost parameter of $\bar{\kappa}_I = 2.50$ basis points (standard error 0.19 basis points) for transition orders, not a price impact coefficient $\bar{\lambda}$ itself. Of course, there is a tight connection between the two concepts. Assuming that orders are broken into pieces and executed at prices which tend to increase along an upward sloping linear supply schedule, total price impact $\bar{\lambda} = 2 \times 2.50$ must be about twice the average impact cost $\bar{\kappa}_I$. Although invariance also has implications for bid–ask spread costs, these costs are negligible for large bets, and hence we ignore them. The implied standard error of $\bar{\lambda}$ is 2×0.19 basis points, about 7% of the estimate 2×2.50.

2.7 A Market Crash Scenario

Suppose both illiquid and liquid assets have annual turnover of 100% over 250 trading days. The illiquid asset is a benchmark stock with $P^* = \$40$ per share, $V^* = 10^6$ shares per day, and $\sigma^* = 0.02$ per day$^{1/2}$. The liquid asset is the entire US stock market, which consists of both the stock index futures market and the cash stock market.

The market has daily dollar volume of about $P \cdot V = \$270$ billion per day, about 6,750 ($= 15^3 \cdot 2$) times the dollar volume of the benchmark stock, and daily returns volatility $\sigma =$ 0.01 per day$^{1/2}$, one-half of stock volatility. Since business time passes at a rate proportional to $(P \cdot V \cdot \sigma/\overline{C})^{2/3}$, the stock market operates about 225 times faster ($= (6,750 \cdot 1/2)^{2/3}$) than the market for the benchmark stock, implying $H = H^*/225$. If bets in the benchmark stock arrive about once every 4 min, bets in the entire market arrive about once per second.

Now let us compare a bet of 25% of daily volume in the benchmark stock with a bet of 25% of daily volume in the market as a whole. For the benchmark stock, the sale would be 250,000 shares worth \$10 million. For the market as a whole, the sale would be slightly less than \$70 billion. Such sales might represent the liquidation of a large institutional position, similar in magnitude to the liquidation of Jérôme Kerviel's rogue trades in 2008, or it might represent many small investors withdrawing equity exposure from index mutual funds or exchange-traded funds (ETFs) over a short period of a few days.

The conventional wisdom and invariance make sharply different predictions about the magnitude of price impacts of these bets.

The conventional wisdom predicts that the price impact of both bets would be miniscule. Since 100% turnover per year implies daily turnover of 0.40% of market capitalization, a bet of 25% of 1 day's volume represents 0.10% of market capitalization. Unit demand elasticity therefore implies a price decline of 10 basis points for both assets, which the market would barely notice.

In contrast, the invariance-implied extrapolation [Equation (13)] leads to very different predictions. For the individual stock, since 1% of daily volume implies a price impact of 5 basis points, linear impact implies that a sale of 25% of daily volume in the benchmark stock has a price impact of 125 basis points. The implied demand elasticity of 0.08 is far smaller than the elasticity of one which represents conventional wisdom and is consistent with the academic literature on the price impact of institutional bets.

For a bet on the entire market, the invariance-implied elasticity is much lower and the price impact is correspondingly greater. Equation (13) implies that since trading activity is higher by a factor of 15^3, price impact would be fifteen times higher if volatility was the same. Since market volatility of 1% per day is half of the daily volatility of 2% for the benchmark stock, price impact is reduced by a factor of 2 from 15 to 7.5 times the price impact of 125 basis points for the individual stock. The price impact of a bet about \$70 billion in the market as a whole is therefore about 937 basis points, similar to the price declines observed when Kerviel's trades were liquidated. The implied demand elasticity for the market as a whole is only 0.01, about 7.5 times smaller than for an individual stock and 100 times smaller than conventional wisdom.

While we disagree with the conventional wisdom that the demand elasticity for financial assets is greater than one, we do not disagree with the greater overall level of price impact for liquid stocks documented by the literature on institutional trades. Instead, by using the invariance hypotheses, we suggest an alternative way of extrapolating price impact for individual stocks to the overall stock market.

Our calculations suggest that the overall stock market is much more fragile than most economists believe. Sudden equity index ETF or mutual fund liquidations of $200 billion over a few days would potentially result in a 30% crash in stock prices, matching the crash of 1987.

3. Examples of Five Market Crashes

We next discuss five crash episodes. The actual price changes during crash events reflect not only sales by particular groups of traders placing large bets but also many other events occurring at the same time, including arrival of news, trading by other traders, and functioning of the trading infrastructure.[5]

3.1 The Stock Market Crash of October 1929

The stock market crash of October 1929 is the most infamous crash in the history of the USA. It became associated with even larger stock price declines from 1930 to 1932, bank runs, and the Great Depression.[6]

The Dow Jones average declined by about 25% during the last week of October 1929 (from 305.85 on October 23 to 230.07 on October 29) and 34% during the last 3 months of 1929 (from 352.57 on September 25 to 234.07 on December 25). These price changes included a 11% drop in the morning on Black Thursday, October 24; a 13% drop on Black Monday, October 28; and another 12% drop on Black Tuesday, October 29.

In the late 1920s, many Americans became heavily invested in a stock market boom. A significant portion of stock investments was made in leveraged margin accounts. Between 1926 and 1929, both the level of margin debt and the level of the Dow Jones average doubled in value. Both the stock market boom and the boom in margin lending came to an abrupt end during the last week of October 1929.

During the week before Black Thursday, October 24, the Dow Jones average fell 9%, including a drop of 6% on Wednesday, October 23, and this led to a self-reinforcing cycle of liquidations of stocks in margin accounts.

To quantify the margin selling which occurred during the last week of October 1929, we follow the previous literature and contemporary market participants by estimating margin selling indirectly from data on broker loans and bank loans collateralized by securities. For the last week of October 1929, we estimate margin selling as $1.181 billion. For the 3 months from September 30, 1929, to December 31, 1929, we estimate total margin selling as $4.348 billion. Details of the estimations are presented in Appendix C.

These liquidations exerted downward price pressure on the stock market. To estimate its magnitude, we treat the 1929 stock market as one market, rather than numerous markets for different stocks, and plug estimates of expected dollar volume and volatility for the entire stock market into Equation (15).

Historical volatility during the month prior to October 1929 was about 2.00% per day. Historical volume was $342.29 million per day in 1929 dollars. Prior to 1935, the volume reported on the ticker did not include "odd-lot" transactions and "stopped-stock"

5 The data for the stock market are from CRSP and the data for index futures are from Bloomberg.

6 Our analysis is based on several documents: Board of Governors of the Federal Reserve System (1927–1931, 1929); Galbraith (1954); Senate Committee on Banking and Currency (1934); Friedman and Schwartz (1963); Smiley and Keehn (1988); and Haney (1932).

transactions, which have been estimated to be equal about 30% of "reported" volume (Board of Governors of the Federal Reserve System, 1943, p. 431). We thus multiply reported volume by 13/10, obtaining an estimate of $444.97 million per day. The margin sales of $1.181 billion during the last week of October were approximately 265% of average daily volume.

Equation (15) implies that margin-related sales of $1.181 billion were expected to trigger a price decline of 46.43%[7]:

$$46.43\% = 1 - \exp\left(-\frac{5.00}{10^4} \cdot \left(\frac{444.97 \cdot 10^6 \cdot 9.42}{40 \cdot 10^6} \right)^{1/3} \cdot \left(\frac{0.0200}{0.02} \right)^{4/3} \cdot \frac{1.181 \cdot 10^9}{(0.01)(444.97 \cdot 10^6)} \right).$$

As a robustness check, Table II reports other estimates using volume and volatility calculated over the preceding m months, with $m = 1$, 2, 3, 4, 6, 12. Invariance predicts price declines ranging from 26.79% to 46.43%, somewhat larger than the actual price change of 25%.[8]

In contrast, since the reduction of broker loans of $1.181 billion was only a very small fraction of the $87.1 billion market capitalization of NYSE issues at the end of September 1929 (Brady Report, p. VIII-13), conventional intuition [Equation (1)] predicts a price change of only 1.36%, much smaller than actual price decline of 25% and about forty times smaller than the magnitude predicted by invariance.

We also make price impact calculations for margin sales of $4.348 billion during the last 3 months of 1929. Conventional wisdom implies a price drop of 4.99%. Invariance implies a much larger price decline ranging from 68.28% to 89.95%, more than the actual price decline of 34% during the last 3 months of 1929 and the price decline of 44% from high point in late September 1929 to low point in mid-November 1929.

3.2 The Market Crash in October 1987

From Wednesday, October 14, 1987, to Tuesday, October 20, 1987, the US equity market suffered the most severe 1-week decline in its history. The Dow Jones index dropped 32% from 2,500 to 1,700; as of noon Tuesday, October 20, the S&P 500 futures prices had dropped about 40% from 312 to 185. On Black Monday alone, October 19, 1987, the Dow Jones index fell 23% and the S&P 500 futures market dropped 29%.

It has long been debated whether the market crash resulted from the sales by institutions implementing portfolio insurance. Portfolio insurance is a trading strategy that replicates put option protection for portfolios by dynamically adjusting stock market exposure in response to market fluctuations. Since portfolio insurers sell stocks when prices fall, the strategy amplifies downward pressure on prices in falling markets. We calculate the price impact of portfolio insurance sales implied by invariance.

7 To convert 1929 dollars to 2005 dollars, we use the GDP deflator, which equates $1 in 1929 to $9.42 in 2005. We use the year 2005 as a benchmark because the estimates of Kyle and Obizhaeva (2016) are based on the sample period 2001–05, with more observations occurring in the latter part of the sample.

8 The 7% standard error for price impact implies a 2-standard-deviation interval to be $46.43\%(1 \pm 2 \times \%7)$, which is larger than the actual price decline of 25%.

Table II. 1929 stock market crash: implied price impact of margin sales

The table shows the invariance-implied impact of \$1.181 billion of margin sales during the week October 24–30, 1929, and \$4.343 billion of margin sales from September 25 to December 25, along with average daily 1929 dollar volume and daily volatility for $m = 1, 2, 3, 4, 6$, and 12 months preceding October 24, 1929. The conventional wisdom predicts a price decline of 1.36% from October 24 to 29 and 4.99% from September 25 to December 25. The actual price decline was 25% from October 24 to 29 and 34% from September 25 to December 25.

	Months preceding October 24, 1929					
m:	1	2	3	4	6	12
ADV (1929 \$M)	444.97	461.45	436.49	427.20	387.18	390.45
Daily volatility	0.0200	0.0159	0.0145	0.0128	0.0119	0.0111
10/24–10/30 sales (% ADV)	265%	256%	271%	276%	305%	302%
Price impact	46.43%	36.26%	33.75%	29.93%	29.00%	26.79%
9/25–12/25 sales (% ADV)	977%	942%	996%	1,018%	1,123%	1,114%
Price impact	89.95%	80.95%	78.04%	73.01%	71.66%	68.28%

We consider the entire stock market to be one market; this is consistent with the Brady Report. Accordingly, we estimate daily volume as the sum of average daily volume in the futures market and the NYSE for the previous month. Some portfolio insurers abandoned their reliance on the futures markets and switched to selling stocks directly because futures contracts became unusually cheap relative to the cash market. We construct a proxy for sales as the sum of portfolio insurance sales in the futures market and the NYSE from tables in the Brady Report, Figures 13–16, pp. 197–198, obtaining results similar to Gammill and Marsh (1988).

Over the 4 days, October 15, 16, 19, and 20, 1987, portfolio insurers sold S&P 500 futures contracts representing \$10.48 billion in index futures and \$3.27 billion in NYSE stocks. The gross sales amount of \$13.75 billion in futures and stocks is combined for the purpose of analyzing price impact of portfolio insurance sales. Reported values are all 1987 dollars.

In the month prior to the crash, the historical volatility of S&P 500 futures returns was about 1.35% per day, similar to estimates in the Brady Report. The average daily volume in the S&P 500 futures market was equal to \$10.37 billion. The NYSE average daily volume was \$10.20 billion. Portfolio insurance gross sales were equal to about 67% of 1 day's combined volume.

Plugging portfolio insurance gross sales and market parameters into Equation (15) yields a price decline of 16.77%[9]:

$$16.77\% = 1 - \exp\left(-\frac{5.78}{10^4} \cdot \left(\frac{(10.37 + 10.20) \cdot 10^9 \cdot 1.54}{40 \cdot 10^6} \right)^{1/3} \cdot \left(\tfrac{0.0135}{0.02}\right)^{4/3} \cdot \frac{(10.48 + 3.27)}{(0.01)(10.37 + 10.20)} \right).$$

Table III reports other estimates based on historical trading volume and volatility calculated over the preceding m months, with $m = 1, 2, 3, 4, 6, 12$. These estimates range from 11.87% to 16.77%. For robustness, estimates under several alternative assumptions are

9 The GDP deflator of 1.54 converts 1987 dollars to 2005 dollars.

Table III. 1987 stock market crash: effect of portfolio insurance sales

The table shows the invariance-implied impact triggered by portfolio insurers' net sales of S&P 500 futures contracts ($9.51 billion) and NYSE stocks ($1.60 billion), portfolio insurers' gross sales of S&P 500 futures contracts ($10.48 billion) and NYSE stocks ($3.27 billion), portfolio insurers' sales of S&P 500 futures adjusted for purchases of index arbitrageurs ($10.48 billion minus $3.27 billion), and portfolio insurers' sales of NYSE stocks adjusted for sales of index arbitrageurs ($3.27 billion plus $3.27 billion) in 1987 dollars. Average daily dollar volume and daily volatility are based on m months preceding October 14, 1987, with $m = 1, 2, 3, 4, 6$, and 12, both for the S&P 500 futures and CRSP stocks. Conventional wisdom predicts price declines of 0.51% for portfolio insurers' net sells and 0.63% for their gross sells. The actual price decline was 32% for the Dow Jones average and 40% for S&P 500 futures.

	Months preceding October 14, 1987					
m:	1	2	3	4	6	12
S&P 500 Fut ADV (1987 $B)	10.37	11.29	11.13	10.12	10.62	9.85
NYSE ADV (1987 $B)	10.20	10.44	10.48	10.16	10.04	9.70
Daily volatility	0.0135	0.0121	0.0107	0.0102	0.0112	0.0111
Gross sells (% ADV)	66.84%	63.28%	63.65%	67.82%	66.53%	70.33%
Price impact	16.77%	14.18%	12.23%	11.87%	13.20%	13.64%
Price impact of net sales combined	13.78%	11.62%	10.00%	9.71%	10.81%	11.17%
Price impact of S&P 500 sales	14.11%	11.67%	10.10%	10.00%	10.93%	11.45%
Price impact of NYSE sales	13.00%	11.18%	9.56%	9.09%	10.32%	10.53%

presented in Table III. Details are presented in Appendix C. The similarity between predicted and observed price declines is consistent with our hypothesis that heavy selling by portfolio insurers played a dominant role in the crash of October 1987.

The estimates based on conventional wisdom are much smaller. According to the Brady Report, there were 2,257 issues of stocks listed on the NYSE, with a value of $2.2 trillion on December 31, 1986. Conventional wisdom implies that gross sales of $10.48 billion in futures and $3.27 billion in individual stocks, representing 0.63% of shares outstanding in total, would have an impact of only 0.63%. Other alternative models yield estimates not higher than 2%. Citing similar arguments, many experts have rejected the idea that sales of portfolio insurers caused the 1987 market crash.

The invariance-implied estimate of 16.77% is smaller than the price drops of 32% in the cash equity market and 40% in the S&P 500 futures market. The 1987 crash may have been triggered by negative news about anti-takeover legislation as well as trade deficit statistic on October 14. It may have been further aggravated by break-downs in the market mechanism which disrupted index arbitrage relationships, as documented in the Brady Report.

3.3 Trades of George Soros on October 22, 1987

On Thursday, October 22, 1987, just 3 days after the 1987 market crash, George Soros lost $60 million in minutes by selling a large number of S&P 500 futures contracts as prices spiked down 22% at the opening of trading. These sales have been attributed to pessimistic predictions that Robert Prechter made based on the Elliott Wave Theory—a form of technical analysis that looks for recurrent long-term price patterns related to persistent changes in investor sentiment. He pointed out similarities between the 1929 crash and the 1987 crash.

The Commodity Futures Trading Commission, Division of Economic Analysis and Division of Trading and Markets (1988) issued a report describing the events of October 22, 1987, without mentioning Soros by name. At 8:28 a.m. Central Time (CT), approximately 2 min before the opening bell at the NYSE, a customer of a clearing member submitted a 1,200-contract sell order at a limit price of 200, more than 20% below the previous day's close of 258. Over the first minutes of trading, the price dropped to 200, at which point the sell order was executed. At 8:34 a.m., a second identical limit order for 1,200 contracts from the same customer was executed by the same floor broker. These transactions liquidated a long position acquired on the previous day at a loss of about 22% or about \$60 million in 1987 dollars. Within minutes, S&P 500 futures prices rebounded and, over the next 2 h, the market recovered to the levels of the previous day's close. Within days, Soros's Quantum Fund sued the brokerage firm which handled the order, alleging a conspiracy among traders to keep prices artificially low while his sell orders were executed.

Two other events may have exacerbated the decline in prices in the morning of October 22 by increasing the selling pressure. First, when the broker executed the second order, he mistakenly sold 651 more contracts than the order called for. The oversold contracts were taken into the clearing firm's error account and liquidated at a significant loss to the broker. Second, the Commodity Futures Trading Commission, Division of Economic Analysis and Division of Trading and Markets (1988) reports that between 9:34 a.m. and 10:45 a.m. the same clearing firm also entered and filled four large sell orders for another customer—a pension fund—with a total of 2,478 contracts sold at prices ranging from 230 to 241. These additional orders are for almost exactly the same size as Soros's orders. This fact suggests information leakage or coordination regarding the size of these unusually large orders.

We compare the actual price decline of 22% with predictions based on invariance. During the prior month, average daily volatility was 8.63% and average daily volume in the S&P 500 futures market was \$13.52 billion in 1987 dollars. The very high volatility estimate based on crash data is reasonable because market participants expected high volatility to persist. Since Soros's sales started just before the opening of NYSE trading, the arbitrage mechanism which connects stock and futures markets did not have time to work; indeed, futures contracts traded at levels about 20% cheaper than stocks. We thus consider only S&P 500 futures market in this example, not combining it with the market for NYSE stocks.

Each S&P 500 contract had a notional value of 500 times the S&P 500 index. With an S&P 500 level of 258, one contract represented ownership of about \$129,000. Soros' sale of 2,400 contracts, or about \$309.60 million, was equal to 2.29% of average daily volume. Given the prior month estimates, Equation (15) predicts a price decline of 6.27%[10]:

$$6.27\% = 1 - \exp\left(-\frac{5.00}{10^4} \cdot \left(\frac{13.52 \cdot 10^9 \cdot 1.54}{40 \cdot 10^6}\right)^{1/3} \cdot \left(\frac{0.0863}{0.02}\right)^{4/3} \cdot \frac{309.60 \cdot 10^6}{(0.01)(13.52 \cdot 10^9)} \right).$$

10 The GDP deflator of 1.54 converts 1987 dollars to 2005 dollars.

Table IV. October 22, 1987: effect of Soros's trades

The table shows the invariance-implied price impact of (A) Soros's sell order of 2,400 contracts; (B) Soros's sell order of 2,400 contracts plus 651 contracts of error trades (3,051 contracts in total); and (C) Soros's sell order of 2,400 contracts, plus 651 contracts of error trades, plus the sell order of 2,478 contracts by the pension fund (5,529 contracts in total). Calculations use average daily 1987 dollar volume and daily volatility for $m = 1$, 2, 3, 4, 6, and 12 months preceding October 22, 1987, for the S&P 500 futures contracts. Conventional wisdom predicts price declines of 0.01%, 0.02%, and 0.03%, respectively. The actual price decline in the S&P 500 futures market was 22%.

	Months preceding October 22, 1987					
m:	1	2	3	4	6	12
S&P 500 Fut ADV (1987 $B)	13.52	11.72	11.70	10.99	10.75	10.04
Daily volatility	0.0863	0.0622	0.0502	0.0438	0.0365	0.0271
2,400 contracts as % ADV	2.29%	2.64%	2.65%	2.82%	2.88%	3.08%
Price impact A	6.27%	4.50%	3.40%	2.96%	2.36%	1.67%
Price impact B	7.90%	5.68%	4.30%	3.75%	2.99%	2.12%
Price impact C	13.85%	10.06%	7.66%	6.69%	5.36%	3.81%

Table IV presents three sets of estimates based on the historical volume and volatility of S&P 500 futures contracts calculated over the preceding m months, with $m = 1$, 2, 3, 4, 6, 12. Invariance implies (i) price impact of 1.67–6.27% based on 2,400 contracts alone; (ii) price impact of 2.12–7.90% adding 651 error contracts (3,051 contracts in total); and (iii) price impact of 3.81–13.85% adding 2,478 contracts sold by the pension fund (5,529 contracts in total).

The actual price decline of 22% is significantly larger than our estimate. Factors which could have led to large impact include potentially underestimated expected volatility, front-running based on leakage of information about the size of the order, and the peculiarly aggressive execution strategy of placing two limit orders with a limit price of 200, more than 20% below the previous day's close.

Conventional wisdom would imply minuscule price changes. Given the total value of $2.2 trillion of issues listed on the NYSE at the end of 1986, the Soros's order, the erroneous sales, and the sales by the pension fund would be expected to have a combined impact of only 0.03%.

3.4 Liquidation of Kerviel's Rogue Trades in January 2008

On January 24, 2008, Société Générale issued a press release stating that the bank had "uncovered an exceptional fraud." Subsequent reports by Société Générale (2008a, 2008b, 2008c) revealed that rogue trader Jérôme Kerviel had used "unauthorized" trading to place large bets on European stock indices.

Kerviel had established long positions in equity index futures contracts with underlying values of €50 billion: €30 billion on the Euro STOXX 50, €18 billion on DAX, and €2 billion on the FTSE 100. He acquired these naked long positions mostly between January 2 and January 18, then concealed them using fictitious short positions, forged documents, and emails suggesting his positions were hedged. The fall in index values in the first half of

January led to losses on these secret directional bets. Internal investigators became strongly suspicious about the nature of the positions on Friday, January 18.

Société Générale informed the heads of the central bank and the Financial Markets Authority (AMF), the French stock market regulator. The AMF allowed the bank to delay public announcement of the fraud for 3 days so that Kerviel's positions could be liquidated quietly. The head of the central bank also delayed informing the government. After liquidating the positions between Monday, January 21, and Wednesday, January 23, the bank had sustained losses of €6.4 billion which—after subtracting out €1.5 billion profit as of December 31, 2007—were reported as a net loss of €4.9 billion.

As Société Générale liquidated the positions, prices fell all across Europe. The Stoxx Europe Total Market Index (TMI)—which represents all of Western Europe—fell by 9.44% from the close on January 18 to its lowest level on January 21. On Monday, January 21—a bank holiday with muted US financial markets activity—the Fed held an unscheduled meeting of the Federal Open Market Committee (FOMC) via conference call at 6:00 p.m. New York time, several days before its scheduled meeting. At 8:30 a.m. the next day, the Fed announced an unprecedented 75-basis point cut in interest rates, which pushed all prices up and helped Société Générale to liquidate the rest of the position on better terms. We do not know whether Fed officials were aware of Société Générale's situation when this decision was made. According to the Fed's Minutes (Board of Governors of the Federal Reserve System, 2008), published five years later, the purpose of the meeting was to "to update the Committee on financial developments over the weekend and to consider whether we want to take a policy action," but there is no mention of Société Générale. In his memoir, Bernanke (2015, pp. 195–196) said the Fed "had no idea the rogue-trading bombshell was coming," but he mentions (p. 195) "a conference call the morning of January 19 Paris time," during which "senior SocGen managers in Paris and New York had told New York Fed supervisors that the bank would report positive earnings for the fourth quarter, even after taking write-downs on its subprime mortgage exposure."

On the one hand, the surprise early announcement of an interest rate cut could have helped the bank obtain more favorable execution prices on some portion of its trades. On the other hand, January 21 was a bank holiday in the USA; in the previous year, the futures markets had only one-third of the typical volume on days when US markets were closed. Low volume on the bank holiday could have reduced liquidity, making the unwinding of positions more expensive.

Due to significant correlations among European markets, we perform our analysis under the assumption that all European stock and futures markets are one market. Based on data from the World Federation of Exchanges, the seven largest European exchanges by market capitalization in 2008 (NYSE Euronext, London Stock Exchange, Deutsche Börse, BME Spanish Exchanges, SIX Swiss Exchange, NASDAQ OMX Nordic Exchange, and Borsa Italiana) had average daily volume for the month ending January 18, 2008, equal to €69.51 billion.

We also sum average daily volume across the ten most actively traded European equity index futures markets (Euro Stoxx 50, DAX, CAC, IBEX, AEX, Swiss Market Index SMI, FTSE MIB, OMX Stockholm 30, and Stoxx 50 Euro) and find average daily futures volume of €110.98 billion. The total daily volume in both European stock and equity futures markets was equal to €180.49.

Our estimate of expected volatility is 1.10%, the previous month's daily standard deviation of returns for the Stoxx Europe TMI.

Table V. January 2008: effect of liquidating Kerviel's positions

The table shows the invariance-predicted losses of liquidating Kerviel's positions of €50 billion under the assumption that the major European cash and futures markets are integrated, and one Euro is worth \$1.4690. Results are provided based on average daily volume of the major European stock exchanges and index futures as well as daily volatilities of Stoxx Europe TMI, based on m months preceding January 18, 2008, with m = 1, 2, 3, 4, 6, and 12. Conventional wisdom predicts price decline of 0.43%. The actual price decline in the Stoxx Europe TMI was 9.44%.

	Months preceding January 18, 2008					
m:	1	2	3	4	6	12
Stk Mkt ADV (2008 €B)	69.51	66.51	67.37	67.01	66.73	66.32
Fut Mkt ADV (2008 €B)	110.98	114.39	118.05	117.46	127.17	121.26
Daily volatility	0.0110	0.0125	0.0121	0.0117	0.0132	0.0111
Order as % ADV	27.70%	27.64%	26.97%	27.11%	25.79%	26.66%
Price impact	10.79%	12.66%	11.94%	11.53%	12.93%	10.59%
Total losses (2008 €B)	2.77	3.27	3.08	2.97	3.34	2.72
Losses: Adj. A (2008 €B)	5.08	5.58	5.39	5.28	5.65	5.03
Losses: Adj. B (2008 €B)	7.39	7.89	7.70	7.59	7.96	7.34

According to Equation (15), the liquidation of Kerviel's €50 billion position—equal to about 27.70% of the average daily volume in aggregated stock and futures markets—is expected to trigger a price decline of 10.79% across European markets:

$$10.79\% = 1 - \exp\Bigg(-\frac{5.00}{10^4} \cdot \left(\frac{180.49 \cdot 1.4690 \cdot 0.92 \cdot 10^9}{40 \cdot 10^6}\right)^{1/3} \left(\frac{0.0011}{0.02}\right)^{4/3} \frac{50}{(0.01) \cdot 180.49} \Bigg).$$

In this equation, we use an exchange rate of \$1.4690 per Euro to convert Euro volume into US dollar volume and convert 2008 dollars into 2005 dollars to be able to use them in our calibrated formulas.[11]

Table V shows the estimates of price impact based on historical trading volume and volatility calculated over the preceding m months, with $m = 1, 2, 3, 4, 6, 12$. Invariance predicts price changes ranging from 10.59% to 12.93%. The Stoxx TMI index actually fell by 9.44% from the market close of 316.73 on January 18 to its lowest level of 286.82 on January 21.

In contrast, conventional wisdom predicts that sales of €50 billion would have a much smaller price impact of 0.43%, given that it represents less than 1% of the total capitalization of European markets, which was about €11.752 trillion in December 2007, as reported by Federation of European Securities Exchanges.

We also estimate the dollar costs of liquidating the rogue position to range from €2.72 billion to €3.34 billion under different assumptions about expected volume and volatility. Adding mark-to-market losses sustained prior to liquidation leads to estimated losses

11 The GDP deflator of 0.92 converts 2008 dollars into 2005 dollars.

ranging from €5.03 billion to €7.96 billion. These estimates are consistent with officially reported losses of €6.30 billion. Appendix C presents the details.

In explaining the costs of liquidating the positions to shareholders already concerned about the bank's losses on subprime mortgages, bank officials blamed "the very unfavorable market conditions" (see the explanatory note about the exceptional fraud released by Société Générale on January 27). Expressing conventional wisdom, the bank announced that its trades accounted for not more than 8% of turnover on any one of the futures exchanges on which they were conducted and thus did not have a serious market impact. In contrast, when examined through the lens of invariance, the reported losses are almost exactly of the magnitude expected from the price impact of the trades on European stock markets.

3.5 The Flash Crash of May 6, 2010

During the morning of May 6, the S&P 500 declined by 3%. Rumors of a default by Greece had made markets nervous in a context where there was already uncertainty about elections in the UK and an upcoming jobs report in the USA.

During the 5-min interval from 2:40 p.m. to 2:45 p.m. ET, the E-mini S&P 500 futures contract suddenly dropped 5.12% from 1,113 to 1,056. After a pre-programmed circuit breaker built into the CME's Globex electronic trading platform halted trading for 5 s, prices went up 5% over the next 10 min, recovering losses.

After the flash crash, the Staffs of the CFTC and SEC (2010a, 2010b) issued a joint report. They said that an automated execution algorithm sold 75,000 S&P 500 E-mini futures contracts between 2:32 p.m. and 2:51 p.m. on the CME's Globex platform, exactly during the V-shaped flash crash. The E-mini contract represents exposure of fifty times the S&P 500 index, one-tenth the multiple of 500 for the older but otherwise similar contract sold by portfolio insurers in 1987. The program sold S&P 500 exposure of $4.37 billion. The joint report did not mention the name of the seller, but journalists identified the seller as Waddell & Reed.

Many people did not believe that selling 75,000 contracts could have triggered a price decline of 5%, because they implicitly relied on conventional intuition: The $4.37-billion sale represented only 3.75% of the daily volume of about 2,000,000 contracts per day in the S&P 500 E-mini futures market, and its impact was expected to be tiny. Thus, many accused high-frequency traders of failing to provide liquidity as prices collapsed.

We examine whether such an order of 75,000 contracts could have resulted in a flash crash. During the preceding month, the volume in E-mini contracts was about $132 billion per day, and the volume in the stock market was about $161 billion per day; the combined daily volume was $292 billion. The historical daily volatility was 1.07%.

Equation (15) implies that the sales of $4.37 billion—equal to about 3.31% of daily volume in S&P 500 E-mini futures market in the previous month or 1.49% for futures and stock market combined—are expected to trigger a price decline of 0.61%[12]:

12 The GDP deflator of 0.90 converts 2010 dollars into 2005 dollars.

Table VI. Flash crash of May 6, 2010: effect of 75,000 contract futures sale

The table shows the invariance-implied price impact of sales of 75,000 S&P 500 E-mini futures contracts. Calculations are based on average daily volume and volatility of the S&P 500 E-mini futures for the m months preceding May 6, 2010, with $m = 1, 2, 3, 4, 6$, and 12. Conventional wisdom predicts a price decline of 0.03%. The actual price decline in the S&P 500 E-mini futures market was 5.12%.

	Months preceding May 6, 2010					
m:	1	2	3	4	6	12
S&P 500 Fut ADV (2010 \$B)	132.00	107.49	109.54	112.67	100.65	95.49
Stk Mkt ADV (2010 \$B)	161.41	146.50	142.09	143.03	132.58	129.30
Daily volatility	0.0107	0.0085	0.0078	0.0090	0.0089	0.0108
Order as % ADV	1.49%	1.72%	1.73%	1.71%	1.87%	1.94%
Price impact (hist σ)	0.61%	0.49%	0.44%	0.53%	0.55%	0.73%
Price impact ($\sigma = 2\%$)	1.39%	1.52%	1.53%	1.52%	1.61%	1.65%

$$0.61\% = 1 - \exp\Bigg(-\frac{5.00}{10^4} \cdot \left(\frac{(132+161)\cdot 0.90 \cdot 10^9}{40 \cdot 10^6} \right)^{1/3} \cdot \left(\tfrac{0.0107}{0.02}\right)^{4/3} \cdot \frac{75,000 \cdot 50 \cdot 1,164}{0.01 \cdot (132+161) \cdot 10^9} \Bigg).$$

Table VI shows estimates based on historical volume and volatility of S&P 500 E-mini futures contracts calculated over the preceding m months, with $m = 1, 2, 3, 4, 6, 12$. These estimates range from 0.44% to 0.73%. Appendix C provides estimates under the alternative assumptions of 2% volatility and less integrated markets. These estimates tend to be higher, ranging from 0.76% to 2.91%.

The predicted price impact of 0.61% is smaller than the actual decline of 5.12%. As discussed in the next section, unusually fast execution may have significantly increased the temporary impact of these trades and led to rapid rebound in prices afterward. Given that the capitalization of US market was about \$15.077 trillion at the end of 2009, conventional wisdom would predict a price decline of only 0.03%.

4. Policy Implications and Lessons Learned

Application of microstructure invariance to intrinsically infrequent historical episodes requires an exercise in judgement to extract appropriate lessons learned. In some cases, our theory predicts price declines that differ from actual declines. We discuss next factors that can potentially explain these differences. While speculative in nature, our discussion suggests important lessons for policymakers concerned with measuring and predicting crash events of a systemic nature, for asset managers worried about managing market impact costs associated with execution of large trades that might potentially disrupt markets, and for researchers interested in understanding of how financial markets work.

4.1 Price Impact Is Large in Liquid Markets

Market participants often execute large orders by restricting quantities traded to be not more than 5% or 10% of average daily volume over a period of several days. This heuristic strategy is usually believed to be reasonable for individual stocks and thus certainly reasonable for more liquid markets such as markets for stock index futures.

While this strategy is reasonable for trading individual stocks, our analysis shows that it may incur much larger-than-expected costs when implemented in more liquid markets. Invariance implies that the price impact (in volatility units) of trading a given fraction of daily volume is proportional to the cube root of dollar volume and returns volatility. For example, if dollar volume $P \cdot V$ increases by a factor of 1,000—approximately consistent with the difference between the benchmark stock and stock index futures—the market impact of a given fraction of volume in Equation (15) increases by a factor of $(1000)^{1/3} = 10$.

The larger impact in more liquid markets follows from the invariance hypotheses combined with linear price impact.

4.2 Rapid Execution Magnifies Transitory Price Impact

Our analysis sheds some light on how price dynamics in response to large bets depends on the speed of execution. While the long-term impact of bets is likely to depend on their information content, the short-term price dynamics is probably affected by the speed of trading.[13] Speeding up execution exacerbates temporary impact.

If uninformative sell bets are executed faster, prices temporarily fall further and execution prices are worse than with slower execution of the same bet. After order execution is finished, prices revert to the same levels which would have prevailed under slower execution. Extremely rapid bet execution therefore leads to V-shaped price paths, in which prices first plunge sharply and then recover rapidly.

The model of smooth trading of Kyle, Obizhaeva, and Wang (2018) provides a theoretical framework for modeling short-term price reactions to unusually rapid execution of large bets.[14] The model implies that markets interpret extremely rapid, heavy selling as an indication that extremely negative information is about to flow into the market. Prices collapse immediately when a heavy rate of selling is detected. When the expected negative information does not materialize, prices rebound, even though much of the heavy selling continues.

The impact formula (15) contains parameters calibrated using the size of portfolio transition orders but not the speed of execution. Most transitions were executed over a period of a few days, and only the most complex of them were carried out over a period of a few weeks. Executions at a prudent pace were designed to keep impact costs low. Extrapolating estimates from portfolio transitions to sales during crashes implicitly makes the identifying

13 Financial crises eventually followed the crash events of 1929 and liquidation of Kerviel's rogue trades in 2008. Whether margin sales in 1929 or Kerviel's trades in 2008 had information content is a difficult question to frame in a meaningful manner. For example, perhaps Kerviel traded against informed traders who correctly foresaw the impending financial crisis, delaying the incorporation of this information into prices until his own positions were liquidated.

14 The model of smooth trading gives rise in the equilibrium to both endogenous permanent and temporary impacts, λ and κ. In most of the traditional models such as Kyle (1985), price impact is permanent and transaction costs of an informed trader do not depend on the speed of trading as long as he trades continuously.

assumption that crash orders were also executed at the same "natural" or prudent pace as portfolio transition orders.

During 1987 and 2010 flash crashes, larger-than-predicted price declines followed by rapid price recoveries suggest that transitory price impact may have been exacerbated by the extremely rapid rate at which selling took place. According to the Staffs of the CFTC and SEC (2010b), for example, the May 2010 flash crash order was executed extremely rapidly in just 20 min, while in previous months two orders of similar magnitudes had been executed over periods of 5 and 6 h, which is fifteen times slower.[15]

4.3 Policies to Maintain Credit Availability Mitigate Crashes

Some policy responses may help to mitigate the negative effects of crashes. These policies have to aim at easing flow of credit as well as providing funds that will make up the gap in demand and supply. Our analysis suggests that the amount of funds necessary to counteract the shock must be comparable to the size of the shock itself.

A good example is the 1929 market crash, during which price declines were somewhat smaller than predicted by invariance and the crash was well contained until the end of 1929. First, immediately after the initial stock market break on Black Thursday, a group of prominent New York bankers put together an informal fund of about $750 million to buy securities in order to support prices. The fund was similar in size to the margin sales shock of about $1.181 billion. When their decisions were publicized, the sense of panic subsided. Similar actions were undertaken by J.P. Morgan and other bankers after a crash in 1907.

Second, the New York Fed acted prudently in 1929. In the 1920s, bankers and their regulators were aware that if non-bank lenders suddenly withdrew funds from the broker loan market, there would be pressure on the banking system to increase lending to make up the difference. By discouraging banks from lending into the broker loan market prior to the 1929 crash, the New York Fed increased the ability of banks to support it after the stock market crashed. During the last week of October 1929, the New York Fed wisely reversed course and encouraged banks to provide clients with bank loans collateralized by securities. As lenders withdrew funds from the broker loan market, the resulting unprecedented increase in demand deposits at New York banks gave banks plenty of cash for financing increased loans on securities. The New York Fed also encouraged easy credit by purchasing government securities and cutting its discount rate twice. Some brokers cut margins from 40% to 20%.

All of these stabilizing policies smoothed the margin selling out and allowed brokers to liquidate the large positions of under-margined stock investors gradually over 5 weeks, rather than selling collateral off at fire-sale prices over several days. This appears to have helped the market to digest imbalances and reduce temporary price impact, thus avoiding a sudden, brutal bursting of the stock market bubble.

15 The smooth trading model implies that the temporary price impact is linear in the speed of trading. Since selling during the flash crash occurred about fifteen times faster than normal order execution, the model implies transitory price impact to be fifteen times greater than in the case when selling occurs at a "normal" rate, followed by a reversal. Multiplied by 15, our estimates of 0.61% price decline become even larger than the actual decline of 5.12%.

4.4 Effect of Large Bets May Propagate across Integrated Markets

During the 1987 crash, both US markets and many major world markets experienced severe declines, despite the fact that the portfolio insurance selling was confined to the USA. According to Roll (1988), this justifies concluding that portfolio insurance did not trigger the 1987 crash.

Common patterns across markets were also documented during liquidation of Kerviel's positions in January 2008. Even markets where Société Générale did not liquidate positions had very similar performance. The bank thus argued that its own impact on prices was limited and large price declines in multiple markets had to be attributed to other factors.

In contrast, we believe that financial markets are integrated and heavy selling in one market is likely to affect correlated markets. Due to this connectedness, market impact estimates should take into account how market liquidity is shared across markets for related securities. For the 1987 crash, it supports treating the stock and futures markets as one big market. For Société Générale's trades, it supports aggregating across all European markets rather than focusing only on countries where the bank liquidated positions. At the same time, the question of how to define the boundaries of the market for the purpose of applying invariance hypotheses and predicting price impact in correlated markets remains to be understood.

4.5 "Inefficient" Liquidity Silos May Enhance Stability

There may be a trade-off between efficiency and stability. In markets with less efficient trading arrangements, more capital is required to sustain orderly trading, but this capital also makes the systems more stable during volatile times. Invariance may help to assess the effect of market integration on liquidity.

The inefficiency of financial markets in 1920s may explain their remarkable resilience. During the 1929 crash, the disproportionably large amount of selling related to liquidation of margin loans was more than fifteen times greater than selling during the 1987 crash, as a percentage of GDP, but the price decline was only half as large.[16]

In the 1920s, speculative capital may have been compartmentalized into numerous separate silos. Speculative trading and intermediation associated with underwriting of new stock issues often took place in "pools," which played a role similar to today's hedge funds. The pools were typically dedicated to trading only one stock, and investors in the pools often had close connections to the company whose stock the pool traded; there were no prohibitions against insider trading and no SEC requiring firms to disclose material information to the market. These pools traded actively, used leverage, took short positions, and arbitraged stocks against options, particularly when facilitating distribution of newly issued equity. There were no futures markets or ETFs allowing investors to trade large baskets of stocks easily. When faced with massive liquidations of margin loans, the market may have more speculative capital available to stabilize the situation than in a more "efficiently"

16 The 1987 portfolio insurance trades of $13 billion were equal to only about 0.28% of GDP in that year (1987 GDP was $4.7 trillion); stock prices fell 32%. During the last week of October 1929, the margin-related sales of $1.181 billion were equal to about 1% of GDP (1929 GDP was $104 billion), approximately four times the levels of the 1987 crash; yet stock prices fell by only 25%. Inclusion of additional sales equal to about 3% of GDP in subsequent weeks makes margin selling in 1929 to be more than fifteen times greater than selling during the 1987 crash, as a percentage of GDP.

leveraged system in which institutional investors can spread their capital across markets by trading hundreds of stocks simultaneously.

Invariance-implied estimates would change significantly if instead of being considered as one large market, the stock market in 1929 was thought of as a set of many small, isolated, and thus less liquid markets for individual stocks. One would expect market impact to be much smaller in these less liquid markets for the same market bet. As a hypothetical illustration, suppose the 1929 stock market consisted of 125 separate markets for 125 different stocks, and assume all of them were of the same size and turnover. Compared with one large integrated market, 125 small markets would absorb the same shock $125^{2/3} = 25$ times more slowly and its impact would be $125^{1/3} = 5$ times smaller, as implied by Equations (9) and (15).

4.6 Early Warning Systems May Be Useful and Practical

Some strategies are inherently destabilizing. They have built-in features of negative feedbacks: as prices go down, more selling is required and this pushes prices further down. The more capital is invested into these strategies, the bigger is their potential destabilizing effects on prices. Equipped with quantitative invariance formulas for market impact, one may detect instances when destabilizing strategies become so large that they may put financial markets at risk.

Tuzun (2012) uses invariance to assess the effect of leveraged ETFs on markets. He finds that short ETFs and leveraged long ETFs in financial stocks were close to the tipping point in 2008 and 2009. A price decline of 1% would induce leveraged ETFs to sell about \$1 billion; invariance implies that this imbalance would lead to a further price decline of another 1% and thus potentially trigger a downward spiral.

For some of the five crash events in our article, policymakers or stock market participants also had in hand the information required to quantify the price impact and foresee the systemic risks looming from sudden liquidations of large stock market exposures. Instead, they mistakenly trusted in conventional intuition when assessing the potential magnitudes of price declines, dismissing the danger of meltdowns.

Contrary to the beliefs of some, the market crashes in 1929 and 1987 were not completely unexpected. In both cases, data were publicly available before the events. Data on broker loans were published by the Federal Reserve System and the NYSE before the 1929 crash. Estimates of assets under management by portfolio insurers were available before the 1987 crash.

In both cases, the potential price impact of liquidations was a topic of public discussion among policymakers and market participants. In the months prior to the 1929 stock market crash, brokers were raising margin requirements to protect themselves from a widely discussed collapse in prices which might be induced by rapid unwinding of stock investments financed with margin loans. Market participants watched statistics on broker loans carefully, noting the tendency for total lending in the broker loan market to increase as the stock market rose. Markets were aware that margin account investors were buyers with “weak hands,” likely to be flushed out of their positions by margin calls if prices fell significantly. Discussions about who would buy stocks if a collapse in stock prices forced margin account investors out of their positions resembled similar discussions in 1987 concerning who would take the opposite side of portfolio insurance trades.

The debate about the extent to which portfolio insurance contributed to the 1987 crash started long before the crash itself. On the day the 1987 crash occurred, academics were holding a conference on a topic of potential "market meltdown" induced by portfolio insurance sales. The term "market meltdown," popularized by then NYSE chairman John Phelan, was used in the year or so before the stock market crash to describe a scenario of cascading portfolio insurers' sell orders resulting in severe price declines and posing systemic risks to the economy. Months before the 1987 crash itself, the SEC's Division of Market Regulation, Securities and Exchange Commission (1987)—responding to worries that portfolio insurance had made the market fragile—published a study describing in some detail a potential meltdown scenario induced by portfolio insurance sales, which closely resembled the subsequent crash in October 1987, but the study dismissed the risk of such an event as a remote possibility, accepting the conventional wisdom at the time.

Many market participants were firmly convinced that, given the substantial trading volume in the US equity markets—and especially the index futures market—there was enough liquidity available to accommodate sales of portfolio insurers without any major downward adjustment in stock prices. During hearings before the House Committee on Energy and Commerce (1987) on July 23 prior to the 1987 crash, Hayne E. Leland defended portfolio insurance:

> We indicated that average trading will amount to less than 2% of total stocks and derivatives trading. On some days, however, portfolio insurance trades may be a greater fraction. . . .In the event of a major one-day fall (e.g., 100 points on the Dow Jones Industrial Average), required portfolio insurance trades could amount to \$4 billion. Almost surely this would be spread over 2–3 day period. In such a circumstance, portfolio insurance trades might approximate 9–12% of futures trading, and 3–4% of stock plus derivatives trading.

If regulators had applied simple principles of invariance prior to the 1987 crash, they would have been alarmed by Hayne Leland's projection of potential sales of 4% of stock-plus-futures volume over 3 days in response to a decline in stock prices of about 4% (i.e., 100 points on the Dow Jones average). They would see that the stock market was already close to a tipping point. Historical volume and volatility in July 1987 implied that sales of \$4 billion in response to a 4% price decline would lead to another drop in prices, just slightly smaller than 4%. Absent stabilizing trades by investors trading in an opposite direction, potential portfolio insurance sales were already on the verge of triggering precisely the cascade meltdown scenario practitioners dismissed as a near-zero probability event.

5. Conclusion

Crash-like events continue to occur. The Staffs of the Fed, the CFTC, and SEC (2015) describe the "flash rally" in the US Treasury market on October 15, 2014, during which prices rose rapidly for several minutes and then fell back down. Since the report was not based on audit trail data identifying individual traders, it does not rule out the possibility that the flash rally resulted from rapid buying by one trader. Obizhaeva and Piftankin (2023) describe how the sharp V-shaped devaluation of Russian currency on December 16, 2014, was likely caused by a large multi-billion-dollar bet. The collapse of the Chinese stock market in the summer of 2015 was likely caused by liquidations of margin accounts, as discussed in Bian *et al.* (2018); this crash was in many ways similar to the crash of 1929 in the US market; in both cases, extraordinary steps were taken to stabilize the markets.

Our study offers several practical insights about why stock market crashes happen, how to prevent them if possible, and how to respond to them if they occur. Large price dislocations may occur because of imbalances in demand and supply. Even in highly liquid markets and even if quantities traded are restricted to 5% or 10% of daily volume, execution of large bets may lead to significant price changes. Rapid execution is likely to magnify transitory impact. Heavy selling in one market is likely to spread to economically related markets. Policies aiming at easing flow of credit and providing funds to make up the gap in demand and supply may help to mitigate the adverse effects of crashes, but the amount of funds necessary must be comparable to the size of the shock itself. Early warning systems may potentially help to prepare for crashes and act accordingly to mitigate or avoid them.

Data Availability

The data underlying this article were obtained from public sources. Stock market data were obtained from CRSP. Index futures data were obtained from Bloomberg. Other data were obtained from cited documents.

Appendix A: Implementation Issues

In order to apply microstructure invariance to data on the five crash events, several implementation issues need to be addressed.

First, the volume and volatility inputs into our formulas should not be thought of as parameters of narrowly defined markets of a particular security in which a bet is placed but rather as parameters based on the market as a whole. Securities and futures contracts may share the same fundamentals and have a common factor structure. When a large order moves prices in the S&P 500 futures market, index arbitragers usually insure that prices for the underlying basket of stocks move by about the same amount as well. It is difficult to identify the boundaries of the market. Consistent with the spirit of the Brady Report, we take the admittedly simplified approach of adding together cash and futures volume for three of the four crash events in which stock index futures markets existed. In our analysis of the Soros trades, we ignore cash market volume because his trades were executed so quickly that price pressure in the futures market was not transferred to cash markets, which had not yet opened at the time of his trades.

Second, the spirit of the invariance hypothesis is that volume and volatility inputs into the market impact [Equation (15)] are market expectations prevailing before the bet is placed. Expected volume and expected volatility determine the sizes of bets investors are willing to make and the market depth intermediaries are willing to provide. Different price impact estimates are possible, depending on whether volatility estimates are based on implied volatilities before the crash, implied volatilities during the crash, historical volatilities based on the crash period itself, or historical volatilities based on months of data before the crash. For robustness, we present results based on historical data for different windows prior to the crash event.

Third, it is likely that the price impact of an order is related to the speed with which it is executed. The market impact model [Equation (15)] assumes that orders are executed at a "normal" speed in the relevant units of business time. For example, a very large order in a small stock may be executed over several weeks or even months, while a large order in the stock index futures market may be executed over several hours. The impact model leaves open the possibility that unusually rapid execution of very large orders may increase their temporary price impact, but these effects are hard to quantify properly. We discuss this issue further in Section 4.

Fourth, there have been numerous changes in market mechanisms between 1929 and 2010, including better communications technologies, introduction of electronic handling of orders, a reduction in tick size, and the migration of trading volume from face-to-face trading floors to anonymous electronic platforms. Such changes may have lowered bid–ask spreads, but we believe—in the spirit of Black (1971)—that they have had little effect on market depth, which largely determines the price impact of large bets. We thus apply estimates of market depth based on portfolio transitions during 2001–05 to the entire period 1929–2010.

Fifth, Kyle and Obizhaeva (2016) calibrate both linear and square root impact models consistent with invariance. From an empirical perspective, the square root specification explains price impact for individual stocks somewhat better than the linear model, as consistent with the empirical econophysics literature (Bouchaud, Farmer, and Lillo, 2009), but the linear model explains the price impact of the largest 1% of bets in the most active stocks slightly better than the square root model. Crash events are explained by applying invariance to a linear model. To make this point, "invariance" implicitly assumes a linear impact function in the main part of the article. Due to its concavity, the square root model predicts much smaller price declines during crash events. Appendix C presents these estimates along with estimates based on alternative models.

Appendix B: Estimates for Different Market Impact Models

We compute estimates of predicted price changes based on several alternative models of market impact. Market impact is expected to depend on market characteristics such as market capitalization N, daily share volume V, returns volatility σ, and the corresponding GDP deflator d_{gdp}; unsigned bet size Q; and perhaps the time horizon T over which the bet is executed.

We consider several specifications when calculating the implied magnitudes of simple (non-logged) market impacts $\Delta P/P$:

- The invariance-implied linear model ("Inv-LIN"), discussed in Kyle and Obizhaeva (2016):

$$\frac{\Delta P}{P} = \frac{2 \cdot 2.50}{10^4} \cdot \left(\frac{P \cdot V \cdot d_{\text{gdp}}}{40 \cdot 10^6}\right)^{1/3} \cdot \left(\frac{\sigma}{0.02}\right)^{4/3} \cdot \frac{Q}{(0.01) \cdot V}. \tag{16}$$

- The invariance-implied square-root model ("Inv-SQRT"), discussed in Kyle and Obizhaeva (2016):

$$\frac{\Delta P}{P} = \frac{2 \cdot 12.08}{10^4} \cdot \left(\frac{\sigma}{0.02}\right) \cdot \left(\frac{Q}{(0.01) \cdot V}\right)^{1/2}. \tag{17}$$

- The conventional model ("Conv-N"), based on market capitalization:

$$\frac{\Delta P}{P} = \frac{Q}{N}. \tag{18}$$

- The Conv-V, based on daily volume:

$$\frac{\Delta P}{P} = \frac{Q}{250 \cdot V}. \tag{19}$$

- The Barra model, discussed in Torre and Ferrari (1999) and Grinold and Kahn (2000):

$$\frac{\Delta P}{P} = \sigma \cdot \left(\frac{Q}{V}\right)^{1/2}. \tag{20}$$

- Almgren–Chriss ("AC") model, discussed in Almgren *et al.* (2005):

$$\frac{\Delta P}{P} = 0.314 \cdot \sigma \cdot \frac{Q}{V} \cdot \left(\frac{N}{V}\right)^{1/4} + 2 \cdot 0.142 \cdot \sigma \cdot \left(\frac{Q}{V \cdot T}\right)^{3/5}. \tag{21}$$

- Frazzini–Israel–Moskowitz ("FIM") model, discussed in Frazzini, Israel, and Moskowitz (2018) in Table AI, Column (9):

$$\frac{\Delta P}{P} = \left(-0.2 \cdot \ln(1 + N \cdot 10^{-9} \cdot d_{\text{gdp}}) + 0.35 \cdot \frac{Q}{0.01 \cdot V} + 9.32 \cdot \left(\frac{Q}{0.01 \cdot V}\right)^{1/2} + 0.13 \cdot \sigma \cdot \sqrt{252} \cdot 100\right) \cdot \frac{2}{10^4}. \tag{22}$$

In the last two models, the estimates are multiplied by a factor of 2 to convert transaction costs estimates to price impact estimates.

The AC model [Equation (21)] explicitly depends on the execution horizon T. For the 1929 crash, we assume selling occurred over 5 days ($T = 5$). For the 1987 crash, we assume selling occurred over 4 days ($T = 4$). For Soros' trades, we assume selling occurred over 6 min from 8:28 a.m. to 8:34 a.m. ($T = 6/420$). For the liquidation of Kerviel's trades, we assume selling occurred over 3 days ($T = 3$). For the flash crash of 2008, we assume selling occurred over about 20 min, or 1/20 of a day ($T = 1/20$).

Panel A of Table AI presents impact estimates based on six impact models for percentage market impact along with actual price declines for the five crashes. First, all estimates are much lower than actual price declines, except for the Inv-LIN estimates. Second, the Conv-N and Conv-V estimates based on the conventional intuition usually generate the smallest estimates among models. Third, calibrated on the sample of institutional transactions, the Barra, AC, and FIM estimates are all similar in magnitude; they are slightly larger than conventional estimates but still much lower than the actual price declines. Fourth, the AC estimate is significantly larger than other alternative estimates for the Soros bet because this estimate explicitly accounts for the very short execution horizon of this bet.

For all five crashes, the Inv-SQRT estimates are quantitatively similar to the Barra estimates. Due to its concavity, the square-root models predict much smaller price declines than the linear model. Thus, invariance alone does not explain magnitudes of price declines during crash events; instead, crash events are explained by applying invariance to a linear model.

Panel B of Table AI presents impact estimates based on six impact models for log-percentage market impact $\Delta \ln P$ along with actual price declines for the five crashes. These estimates are obtained from models (16)–(22), where $\Delta P/P$ on the left-hand side of these equations is replaced with $\Delta \ln P$. The Inv-LIN and the Conv-N based on market capitalization for log-impact are the two models discussed on detail in the main part of our article.

The estimates based on log-returns are smaller than the estimates based on simple returns, but this difference is negligible for most models. The only exception is the Inv-LIN

Table AI. Alternative models

The table presents actual price declines for five market crashes along with price declines as implied by seven price impact models: the Inv-LIN, the Inv-SQRT, the Conv-N based on market capitalization, the Conv-N based on daily volume, the Barra model ("Barra"), AC model, and FIM model. Panel A shows the estimates for models with simple returns $\Delta P/P$ and panel B shows the estimates for models with log returns $\Delta \ln P$.

Panel A: Simple percentage impact $\Delta P/P$

Crash Event	Actual (%)	Inv-LIN (%)	Inv-SQRT (%)	Conv-N (%)	Conv-V (%)	Barra (%)	AC (%)	FIM (%)
1929 market crash	25.00	62.56	3.94	1.36	1.06	3.26	6.62	4.95
1987 market crash	32.00	18.31	1.33	0.63	0.27	1.10	1.04	2.02
1987 Soros's trades	22.00	6.47	1.58	0.01	0.01	1.31	3.47	0.62
2008 SocGén trades	9.44	11.40	0.70	0.43	0.11	0.58	0.35	1.18
2010 flash crash	5.12	0.61	0.16	0.03	0.01	0.13	0.16	0.24

Panel B: Log-percentage impact $\Delta \ln P$

Crash Event	Actual (%)	Inv–LIN (%)	Inv–SQRT (%)	Conv–N (%)	Conv–V (%)	Barra (%)	AC (%)	FIM (%)
1929 market crash	25.00	46.43	3.86	1.36	1.06	3.21	6.41	4.83
1987 market crash	32.00	16.77	1.32	0.63	0.27	1.10	1.04	1.99
1987 Soros's trades	22.00	6.27	1.57	0.01	0.01	1.30	3.41	0.62
2008 SocGén trades	9.44	10.79	0.70	0.43	0.11	0.58	0.35	1.17
2010 flash crash	5.12	0.61	0.16	0.03	0.01	0.13	0.16	0.24

model, for which large estimates based on the simple return are reduced when log-returns are used instead; the biggest difference is observed for the 1929 crash, for which the simple-return model implies price decline of 63% and the log-return model implies price decline of only 46%.

Appendix C: Estimation Details

Estimation Details for the Crash of 1929

A significant portion of stock investments in the late 1920s was made in leveraged margin accounts. To finance their leveraged purchases of stocks, individuals and non-financial corporations relied either on bank loans collateralized by securities or on margin account loans at brokerage firms. When investors borrowed through margin accounts at brokerage firms, the brokerage firms financed only a modest portion of the loans with credit balances from other customers. To finance the rest, brokerage firms pooled securities pledged as collateral by customers under the name of the brokerage firm (in "street name") and then re-hypothecated these pools by using them as collateral for broker loans. The broker loan market of the late 1920s resembled the shadow banking system of the early 2000s in its lack of regulation, perceived safety, and the large fraction of overnight or very short maturity loans.

The broker loan market was controversial during the 1920s, just as the shadow banking system was controversial during the period surrounding the financial crisis of 2008–09.

Some thought the broker loan market should be tightly controlled to limit speculative trading in the stock market on the grounds that lending to finance stock market speculation diverted capital away from more productive uses in the real economy. Others thought it was impractical to control lending in the market because the shadow bank lenders would find ways around restrictions and lend money anyway. The New York Fed chose to discourage New York banks from lending money against stock market collateral. As a result, loans to brokers by New York banks declined after reaching a peak in 1927.

Attracted by the high interest rates on broker loans—typically 300 basis points or more higher than loans on otherwise similar money market instruments—non-New York banks and non-bank lenders continued to supply capital to the broker loan market. Many of these loans were arranged by the New York banks; sometimes, non-bank lenders bypassed the banking system entirely, making loans directly to brokerage firms.

Investment trusts (similar to closed end mutual funds) placed a large fraction of the newly raised equity into the broker loan market rather than buying expensive common stocks. Corporations, flush with cash from growing earnings and proceeds of securities issuance, invested a large portion of these funds in the broker loan market rather than in new plant and equipment.

To quantify the margin selling which occurred during the last week of October 1929, we follow the previous literature and contemporary market participants by estimating margin selling indirectly from data on broker loans and bank loans collateralized by securities.

In the 1920s, data on broker loans came from two sources. First, the Fed collected weekly broker loan data from reporting member banks in New York City supplying the funds or arranging loans for others. Second, the New York Stock Exchange collected monthly broker loan data based on demand for loans by NYSE member firms. The broker loan data reported by the New York Stock Exchange include broker loans which non-banks made directly to brokerage firms without using banks as intermediaries; such loans bypassed the Fed's reporting system. Since loans unreported to the Fed fluctuated significantly around the 1929 stock market crash, we rely relatively heavily on the NYSE numbers in our analysis below but also pay careful attention to the weekly dynamics of the Fed series for measuring selling pressure during the last week of October 1929.

We calculate weekly proxies for margin sales as follows. (1) We difference the weekly Fed series to construct weekly changes. (2) We interpolate the monthly NYSE series to construct a weekly series by assuming that these loans changed at a constant rate within each month, except for October 1929. For October 1929, the Fed series shows little change, except for the last week, and we therefore assume that the entire monthly change in the NYSE series represents unreported changes in broker loans which occurred during the last week of October 1929. (3) Finally, we add changes in bank loans collateralized by securities to take into account the fact that some changes in broker loans do not represent margin sales because they were converted into bank loans collateralized by securities. The last adjustment also has a significant effect because there was an unprecedented increase in banks loans collateralized by securities during the last week of October 1929, followed by offsetting reductions during November.

Figure AI shows the weekly levels of the Fed's broker loan series and the monthly levels of the NYSE broker loan series. Two versions of each series are plotted, one with bank loans collateralized by securities added and one without ("Fed Broker Loans," "Fed Broker Loans + Bank Loans," "NYSE Broker Loans," "NYSE Broker Loans + Bank Loans"). The figure also shows the level of the Dow Jones Industrial Average from 1926 to 1930. The

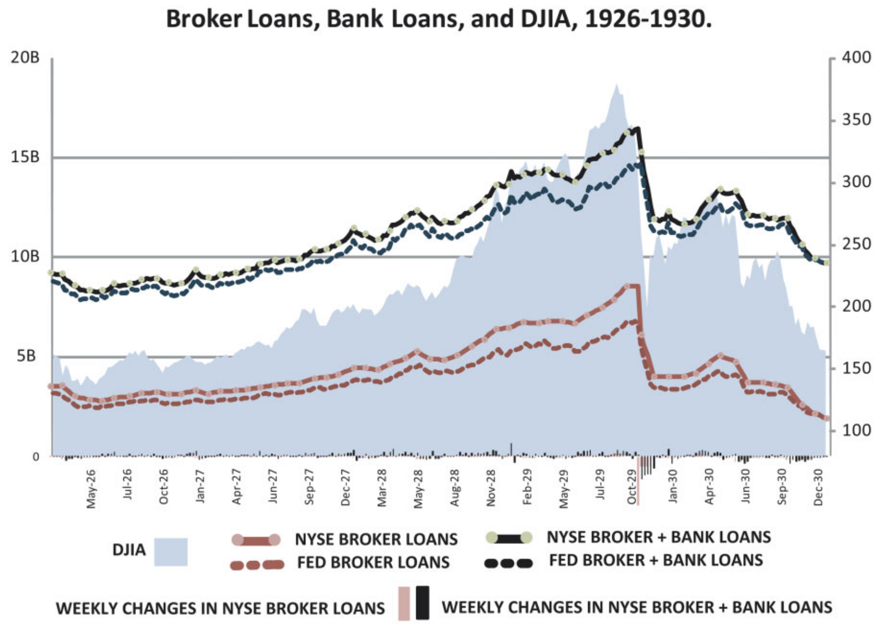

Figure AI. Broker loans and the 1929 market crash. The figure shows weekly dynamics of seven variables from January 1926 to December 1930: NYSE broker loans (red solid line), Fed broker loans (red dashed line), the sum of NYSE broker loans and bank loans (black solid line), the sum of Fed broker loans and bank loans (black dashed line), changes in NYSE broker loans (red bars), changes in the sum of NYSE broker loans and bank loans (black bars), and the Dow Jones average (in blue). Monthly levels of NYSE broker loans are marked with solid dots. Weekly levels of NYSE broker loans are obtained using a linear interpolation from monthly data, except for October 1929, when all changes in NYSE broker loans are assumed to occur during the last week.

time series on both broker loans and stock prices follow similar patterns, rising steadily from 1926 to October 1929 and then suddenly collapsing. According to Fed data, broker loans rose from \$3.141 billion at the beginning of 1926 to \$6.804 billion at the beginning of October 1929. According to NYSE data, the broker loan market rose from \$3.513 billion to \$8.549 billion during the same period. As more and more non-banks were getting involved in the broker loan market, the difference between NYSE broker loans and Fed broker loans steadily increased until the last week of October 1929, when non-bank firms pulled their money out of the broker loan market and the difference suddenly shrank.

During the period 1926–30, weekly changes in broker loans were typically small and often changed sign, as shown in the tiny bars at the bottom of Figure AI. Starting with the last week of October 1929, there were five consecutive weeks of large negative changes, almost twenty times larger than changes during preceding weeks. This de-leveraging erased the increase in broker loans which had occurred during the first 9 months of the year.

For the last week of October 1929, we estimate margin selling as \$1.181 billion (the difference between the estimated reduction in broker loans of \$2.440 billion from \$8.549 billion to \$6.109 billion and an increase in bank loans on securities of \$1.259 billion from \$7.920 billion to \$9.179 billion). For the 3 months from September 30, 1929, to December 31, 1929, we estimate margin selling as \$4.348 billion (the difference between the reduction in NYSE broker loans of \$4.559 billion from \$8.549 billion to \$3.990 billion and an increase in bank loans on securities of \$0.211 billion from \$7.720 billion to \$7.931 billion).

Estimation Details for the Crash of 1987

Along with our main estimates in Table III, we present several other estimates for robustness. First, some of the market participants classified as portfolio insurers in the Brady Report abandoned their portfolio insurance strategies as prices crashed and switch to buying securities. Even though we believe that for the purpose of analyzing the price impact of portfolio insurance sales it is better to use the gross sales amount, we also report estimates for net sales of \$11.11 billion of futures contracts and stocks combined (\$9.51 billion in futures and \$1.60 billion in stocks). Their predicted impact ranges from 9.71% to 13.78%.

Second, we show implied estimates if we treat markets for futures contracts and NYSE stocks separately. To avoid radically different price impacts in two markets, we adjust quantities sold in both markets by the NYSE's estimate of net NYSE index-arbitrage sales of \$3.27 billion (Brady Report, Figures 13 and 14). We add this number to portfolio insurance sales in NYSE stocks and subtract the same amount from portfolio insurance sales in the futures market because arbitrageurs transferred some price pressure from futures to stocks. This results in net sales of \$7.21 billion in the futures market with impact ranging from 10.00% to 14.11% and \$6.54 billion in NYSE stocks with impact ranging from 9.09% to 13.00%. The fact that index-arbitrage sales make price impact estimates similar in both markets is consistent with the interpretation that portfolio insurance sales were indeed driving price dynamics in both markets.

Estimation Details for Liquidation of Kerviel's Trades in 2008

We also examine whether implied cost estimates are consistent with officially reported losses of €6.30 billion. We assume that average impact cost is equal to half of predicted price impact since—assuming no leakage of information about the trades—a trader can theoretically walk the demand curve, trading only the last contracts at the worst expected prices. Accounting for compounding, invariance predicts that the total cost of unwinding Kerviel's position is equal to 5.55% of the initial €50 billion position, that is, €2.77 billion.

Officially reported losses also include mark-to-market losses sustained by hidden naked long positions as markets fell from the end of the previous reporting period on December 31, 2007, to the decision to liquidate the positions when the market re-opened after January 18, 2008. From December 28, 2007, to January 18, 2008, the Euro STOXX 50 fell by 9.18%, DAX futures fell by 9.40%, and FTSE futures fell by 8.68%. If we assume that Kerviel held a constant long position from December 31, 2007, to January 18, 2008, then these positions would have sustained €4.62 billion in mark-to-market losses during that period. Société Générale reported, however, that Kerviel acquired his hidden long position gradually over the month of January. If we assume that Kerviel acquired his position gradually by purchasing equal quantities of futures contracts at each lower tick level from the end-of-year 2007 close to January 18 close, we estimate that such positions would be under water by only half as much, that is, €2.31 billion, at the close of January 18.

Table V reports that the estimated market impact costs of liquidating the rogue position range from €2.72 billion to €3.34 billion under different assumptions about expected volume and volatility. Adding mark-to-market losses sustained prior to liquidation leads to estimated losses ranging (i) from €5.03 billion to €5.65 billion if positions were acquired gradually and (ii) from €7.34 billion to €7.96 billion if positions were held from the end of

2007. These estimates are similar in magnitude to losses of €6.30 billion reported by the bank.

As a robustness check, we also estimate market impact under the assumption that the Euro STOXX 50, the DAX, and the FTSE 100 futures markets are distinct markets, not components of one bigger market.

In the month preceding January 18, 2008, historical volatility per day was 98 basis points for futures on the Euro STOXX 50, 100 basis points for futures on the DAX, and 109 basis points for futures on the FTSE 100. Average daily volume was €55.19 billion for Euro STOXX 50 futures, €32.40 billion for DAX futures, and £7.34 billion for FTSE 100 futures. Kerviel's positions of €30 billion in Euro STOXX 50 futures, €18 billion in DAX futures, and €2 billion in FTSE 100 futures represented about 54%, 56%, and 20% of daily trading volume in these contracts, respectively. We use an exchange rate of €1.3440 for £1 on January 17.

Our calculations estimate a price impact of 12.08% for liquidation of Kerviel's position, 10.77% for liquidation of his DAX futures position, and 4.12% for liquidation of his FTSE futures position. Indeed, from the close on January 18 to the close on January 23, Euro STOXX 50 futures fell by 10.50%, DAX futures fell by 11.91%, and FTSE 100 futures fell by 4.65%. Note that from the close on January 18 to the lowest point during January 21–23, Euro STOXX 50 futures fell by 11.67%, DAX futures fell by 12.71%, and FTSE 100 futures fell by 9.54%. The similarity of actual price declines for the STOXX 50, DAX, and FTSE suggests substantial integration of European markets, consistent with our strategy of thinking about them as one market.

From the close on January 18 to low points on January 22, the Spanish IBEX 35, the Italian FTSE MIB, the Swedish OMX, the French CAC 40, the Dutch AEX, and the Swiss Market Index fell by 12.99%, 10.11%, 8.63%, 11.53%, 10.80%, and 9.63%, respectively. By January 24, all of these markets had largely reversed these losses. Euro Stoxx 50 and FTSE reversed losses as well, but DAX recovered only partially. Large price declines in markets where Kerviel did not hold positions suggest that the markets are well integrated as well.

Estimation Details for the Flash Crash of May 6, 2010

Since the price drop in the morning may have reset market expectations about volatility, as a robustness check, we also report results for expected volatility of 2.00% per day; they range from 1.39% to 1.65%.

If we do not treat the cash market and the futures market as one market but focus only on the futures market, then the estimates range from 0.76% to 1.29% for historical volatility and from 2.35% to 2.91% for volatility of 2%.

Appendix D: The Frequency of Market Crashes

Market microstructure invariance can be used to quantify the frequency of crash events, including both the size of selling pressure and the resulting price impact.

Using portfolio transitions orders as proxies for bets, Kyle and Obizhaeva (2016) find that the invariant distributions of buy and sell bet sizes can be closely approximated by a log-normal. The distribution of unsigned bet size $\tilde{X}$ of a stock with expected daily volume of $P \cdot V$ dollars and expected daily returns volatility σ can be approximated as a log-normal

$$\ln\left(\frac{\tilde{X}}{V}\right) = -5.71 - \frac{2}{3} \cdot \ln\left(\frac{\sigma \cdot P \cdot V}{(0.02)(40)(10^6)}\right) + \sqrt{2.53} \cdot \tilde{Z}, \tag{23}$$

where $\tilde{Z} \sim \mathcal{N}(0,1)$. Under the assumption that there is one unit of intermediation trade volume for every bet, the bet arrival rate γ per day is given by

$$\ln(\gamma) = \ln(85) + \frac{2}{3} \cdot \ln\left(\frac{\sigma \cdot P \cdot V}{(0.02)(40)(10^6)}\right). \tag{24}$$

These equations have the following implications for a benchmark stock with dollar volume of \$40 million per day and volatility 2% per day$^{1/2}$. The estimated mean of –5.71 implies a median bet size of approximately \$132, 500, or 0.33% of daily volume. The estimated log-variance of 2.53 implies that a one-standard-deviation increase in bet size is a factor of about 4.91. The implied average bet size is \$469,500 and a four-standard-deviation bet is about \$77 million, or 1.17% and 192% of daily volume, respectively [$0.33\% \cdot \exp(2.53/2)$ and $0.33\% \cdot \exp(2.53 \cdot 4)$]. There are 85 bets per day. The standard deviation of daily order imbalances is equal to 38% of daily volume [$85^{1/2} \exp(-5.71 + 2.53)$]. Half the variance in returns results from fewer than 0.10% of bets and suggests significant kurtosis in returns.

Now let us extrapolate these estimates to the entire market, where volume is the sum of the volume of CME S&P 500 futures contracts and all individual stocks. Using convenient round numbers based on the 2010 flash crash, the volume for the entire market is about \$270 billion per day or 6,750 times the volume of a benchmark stock. The volatility of the index is about 1% per day or half of 2% volatility of a benchmark stock. With 6,750 conveniently equal to $15^3 \cdot 2$, invariance implies that market volume consists of 19,125 bets ($85 \cdot 15^2$) with the median bet of about \$4 million (\$$132,500 \cdot 15 \cdot 2$) or 0.0014% of daily volume. The implied average bet size is \$14 million or 0.0052% of daily volume, and a four-standard-deviation bet is \$2.310 billion (\$$469,500 \cdot 15 \cdot 2$ and \$$77 \cdot 10^6 \cdot 15 \cdot 2$) or 0.86% of daily volume. The implied standard deviation of cumulative order imbalances is 2.55% of daily volume (38%/15).

Equations (23) and (24) can be used to predict how frequently crash events occur. The three large crash events—the 1929 crash, the 1987 crash, and the 2008 Société Générale trades—are much rarer events than the two smaller crashes—the 1987 Soros trades and the 2010 flash crash.

We estimate the 1929 crash, the 1987 crash, and the 2008 liquidation of Kerviel's positions to be 6.15, 5.97, and 6.19 standard deviation bet events, respectively. Given corresponding estimated bet arrival rates of 1,887, 5,606, and 19,059 bets per day, such events would be expected to occur only once every 5,516, 597, and 674 years, respectively. Obviously, either the far right tail of the distribution estimated from portfolio transitions is fatter than a log-normal or the log-variance estimated from portfolio transition data is too small. In the far right tail of the distribution of the log-size of portfolio transition orders in the most actively traded stocks, Kyle and Obizhaeva (2016) do observe a larger number observations than implied by a normal distribution. It is also possible that portfolio transition orders are not representative of bets in general. If the true standard deviation of log bet size is 10% larger than implied by portfolio transition orders, then 6.0 standard deviation events become 5.4 standard deviation events, which are expected to occur about thirty-four times more frequently.

We estimate the 1987 Soros trades and the 2010 flash crash trades to be 4.45- and 4.63-standard-deviation bet events, respectively. Given estimated bet arrival rates of 14,579 bets and 29,012 bets per day, respectively, bets of this size are expected to occur multiple times per year. We believe it likely that large bets of this magnitude do indeed occur multiple times per year, but execution of such large bets typically does not lead to flash crashes because such large bets would normally be executed more slowly and therefore have less transitory price impact.

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